Robust Water Resources Management by Using Fuzzy-Stochastic OWA Operator

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Abstract

All realistic multi criteria decision making (MCDM) problems face various kinds of uncertainty. One of these uncertain parameters is the optimism degree of the Decision Maker (DM), which has an important effect on the results. Fuzzy linguistic quantifiers will be used to obtain the values of this parameter and then, it will be assumed to have stochastic nature. A new approach will be introduced for the fuzzy-stochastic modeling of MCDM problems by merging the stochastic and fuzzy approaches into an improved OWA operator.

The results of the new approach give the expected value and the variance of the combined goodness measure for each alternative. In order to combine these two characteristics a composite goodness measure will be introduced. The theoretical results will be illustrated in the Sefidrud watershed management problem. By using this measure more sensitive decisions are given to the stakeholders whose optimism degrees are different than that of the decision maker.

Keywords: Multi criteria decision making; Watershed management; Ordered weighted averaging; Fuzzy linguistic quantifiers; Stochastic uncertainty;

Introduction

In real decision making problems multiple criteria have to be usually considered. The resulting MCDM models face different kinds of uncertainty, which generally arise from two sources: stochastic uncertainty related to environmental, economic or technical data, and fuzzy uncertainty related to subjective judgments and the characteristics of the DM. Earlier works on MCDM models under uncertainty did not utilize the mixture of the different types of uncertainty for a given problem and they assumed the existence of either stochastic or fuzzy uncertainty.

Stochastic nature: One of the benchmarking works in stochastic MCDM is PORTRADE introduced by Goicoechea et al. (1982). This method requires an assessment of the DM's multi-attribute utility function and uses chance constraints to find a tradeoff between expected levels of the objective functions and their respective probabilities of achieving desired levels. STRANGE, introduced by Teghem et al. (1986) is a progressive articulation scenario-based technique that assumes given discrete sets of model parameters, each with its own subjective probability of occurrence. Stancu-Minasian (1984) gives a survey of earlier methodologies developed in stochastic MCDM. Changchit and Terrell (1993) proposed a model for stochastic goal programming to be used in water resources management. Ben Abdelaziz et al. (1999) analyzed the efficiency of stochastic MCDM problems when the random variables are discrete. Solutions of stochastic MCDM problems are usually obtained by using two transformations: the

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stochastic problem is transformed first into its equivalent deterministic problem, and then the MCDM problem is rewritten into a single objective optimization problem. Caballero et al. (2004) examined whether the order in which these two transformations are applied influences the obtained efficient solution. More recently, Hahn (2006) developed a stochastic formulation of the Analytic Hierarchy Process (AHP) based on Bayesian categorical data models. Lahdelma and Salminen (2006) introduced the SMAA-D method, which is a combination of data envelopment analysis and stochastic multicriteria acceptability analysis to handle uncertain and imprecise data to provide stochastic efficiency measures.

Fuzzy nature: Fuzzy set theory has been extended into decision making by Bellman and Zadeh (1970). Zimmermann (1978) showed the consequences of using different ways of combining individual objective functions in order to determine an optimal compromise solution in fuzzy environment. The works of (Chen and Hwang, 1991, Ribeiro, 1996, and Wang, 2000) have comprehensive literature reviews on fuzzy MCDM methodology. In recent years, Triantaphyllou and Lin (1996) evaluated five fuzzy MCDM methods: fuzzy SAW model, fuzzy weighted product model, fuzzy AHP, revised fuzzy AHP and fuzzy TOPSIS. Bender and Simonovic (2000) developed fuzzy compromise programming and applied it to solve a water resource management problem. Chen (2000) extended the fuzzy TOPSIS method into group decision making when the evaluations of the alternatives versus the criteria use linguistic variables. Shamsuzzaman et al. (2003) presented a computational approach that combines fuzzy sets with the AHP method. Xu and Chen (2007) developed an interactive method for group MCDM in fuzzy environment for situations where only partial information is available about criteria weights.

Other authors have compared stochastic and fuzzy MCDM methodologies. The main difference between stochastic and fuzzy MCDM is that stochastic MCDM counts all ways to accomplish a task but fuzzy MCDM looks for a best way to do the job (Buckley, 1990). Slowinski and Teghem (1990) compared these two approaches and defined a guideline for method selection based on the following four criteria: the type of information at one's disposal, the way in which the DM perceives and defines the imprecision of the data, the personality of the DM collaborating with the analyst, and the information that the DM wishes to obtain. Roubens and Teghem (1991) stated that fuzzy MCDM models are better than stochastic models in modeling the uncertainty of the constraints and reversely, stochastic MCDM models are more precise in modeling the parameters and the coefficients of the objective function. Solving a fuzzy MCDM problem is usually easier than solving a stochastic MCDM problem (Inuiguichi and Ramik, 2000).

Fuzzy-Stochastic nature: In recent years, an increasing attention has been given to the solution of MCDM problems involving both stochastic and fuzzy uncertainties. The most important such works are listed in Table 1, including their methods and fields of application.

All earlier works listed in Table 1, consider continuous multi-objective programming problems and not problems with distinct alternatives. In this paper a fuzzy-stochastic MCDM model will be introduced to obtain and select optimal alternatives. This model is called the Fuzzy-Stochastic-Precise Ordered Weighted Averaging (FSPOWA) method, which consists of the following four steps:

Step 1- The combined goodness measure for each alternative is produced by the OWA operator.

Step 2- The optimism degree of the DM is obtained by using fuzzy linguistic quantifiers.

Step 3- The optimism degree is assumed to be uncertain, and a PDF is considered for its value.

Step 4- A robust decision is obtained by combining the expected value and variance of the combined goodness measure for each alternative.

In this paper the details of this new approach will be introduced, and this new methodology will be applied in a real case study. The paper develops as follows. In section 2 we will define OWA to create the combined goodness measure for each alternative. Fuzzy linguistic quantifiers will be then used to find the optimism degree of the DM. In section 3 a new method will be introduced to obtain the order weights of the OWA operator. In section 4 the optimism degree will be randomized to model its uncertainty. In section 5 a real case study will be introduced and then a composite goodness measure will be defined and used to find the most satisfying decision alternative.

Table 1- Fuzzy-stochastic multi-criteria decision making methods

Authors	Methods	Applications
Raju and Kumar (1998)	Vagueness and imprecision in the objective function values are quantified by membership functions and the uncertainty in the water inflows by stochastic programming in a fuzzy multi-objective framework.	Irrigation planning
Sinha et al. (2000)	Fuzzy compromise programming in a stochastic linear programming problem.	A numerical example
Mohammed (2000)	Chance constraint method in MCDM problems by assuming uniform distribution of the right hand side coefficients and implying fuzzy satisfaction thresholds.	A numerical example
Mohan and Nguyen (2001)	Stochastic objectives are treated on the basis of extended model and the stochastic constraints as fuzzified chance constraints.	Some numerical examples
Liu et al. (2003)	The model is converted into a deterministic model through transforming imprecise constraints into precise inclusive constraints that correspond to a-cut levels. Parameters in this model are considered both stochastic and fuzzy.	Air quality management
Ben Abdelaziz et al. (2004)	Chance constrained Goal programming including fuzzy numbers.	Water resources management
Sakawa et al. (2004)	Solving LP with random coefficients in the objective function with chance constraint and/or fuzzy satisfying methods.	A numerical example
Das et al. (2004)	Reducing fuzzy-stochastic programming problems to fuzzy non-linear programming problems and then using Zimmermann's technique.	Two-item inventory problem
Bath et al. (2004)	Using an interactive fuzzy satisfying method to decide the multi-objective generation schedule with explicit recognition of stochastic uncertainties.	Electricity generation scheduling
Maqsood et al. (2005)	Interval-parameter fuzzy two stage stochastic programming.	Water resources management
Iskander (2005)	Chance-constraint approach and α-cuts are used to transform the stochastic fuzzy problem into a deterministic-crisp equivalent by using four dominance indices.	A numerical example

OWA operator

There are three basic types of aggregation operators known from the fuzzy set literature: operators for the intersection of fuzzy sets (the Min operator), operators for the union of fuzzy sets (the Max operator), and averaging operators (Eastman et al. 1993). Yager (1988) introduced a special aggregation technique based on the ordered weighted averaging (OWA) operator, which is a common generalization of the three basic aggregation functions (Malczewski, 2006). It satisfies the following condition:

$$F_{Min}(a_1, a_2, ..., a_n) \le F_{OWA}(a_1, a_2, ..., a_n) \le F_{Max}(a_1, a_2, ..., a_n) \quad (1)$$

Since its introduction, the OWA operator has been used in many fields including neural networks, database systems, fuzzy logic controllers, expert systems, market research, linguistic quantified propositions, mathematical programming, lossless image compression, and also in solving MCDM problems (Xu and Da, 2003).

An n-dimensional OWA operator assigns a combined measure for each alternative in a MCDM problem as

$$F_{OWA}(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j = w_1 b_1 + w_2 b_2 + ... + w_n b_n \ , \quad (2)$$

where $F_{OWA} : R^n \mapsto R$, and b_j is the jth largest element in the set of inputs $\{a_j\}$. The coefficients, w_j, are the order weights such that $w_j \ge 0$ and $\sum_{j=1}^{n} w_j = 1$. In this study, a_j represents the distance between the evaluations of an alternative and the components of the nadir. The objective is to maximize F_{OWA}.

Notice that the components of the input vector have been ordered before multiplying them by the order weights. The OWA method has a large variety by the different selections of the order weights. The order weights depend on the optimism degree (well known as Orness degree) of the DM (Yager, 2002). The larger the weights are at the beginning of the weight vector, the higher is the optimism degree (risk acceptance). The optimism degree of the DM is a measure defined to vary from zero (for very risk aversion DM) up to one (for very risk prone DM). The neutral DM's optimism degree can be presented by 0.5. Based on these weights, Yager (1988) defined the optimism degree, θ, as follow:

$$\theta = \frac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j \ . \quad (3)$$

Xu (2005) gives a general overview of the different methods for determining the order weights. We will next introduce an improved OWA method, called POWA.

Fuzzy Stochastic Precise OWA

In natural languages we use many linguistic terms such as *most, few, many*, and *about half.* Zadeh (1983) called them linguistic quantifiers and suggested to model these linguistic quantifiers by using fuzzy sets. They are used to characterize the aggregation inputs in an OWA operator. In this paper these linguistic quantifiers are modeled by Regular Increasing Monotonic (RIM) quantifiers, Q, in which the more objects are included, the higher is the satisfaction of the DM. That is, $Q(r_1) \ge Q(r_2)$ as $r_1 \ge r_2$.

Yager (1988) suggested obtaining the weights of an n-dimensional OWA operator by relation

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n}), \quad j = 1, 2, ..., n. \quad (4)$$

Notice first that the derivative of the fuzzy linguistic quantifier Q is the following:

$$\frac{dQ}{dr} = \lim_{\Delta r \to 0} \frac{Q(r) - Q(r - \Delta r)}{\Delta r}. \quad (5)$$

In the special case when n is large, we may select $\Delta r = 1/n$, and so

$$\frac{dQ}{dr} \approx \frac{Q(r) - Q(r - 1/n)}{1/n}.$$

Yager (1993) approximated the value of dQ/dr at $r = j/n$ by using equation (4) as

$$\left.\frac{dQ}{dr}\right|_{r=j/n} \approx \frac{Q(j/n)-Q((j-1)/n)}{1/n} = \frac{w_j}{1/n},$$

that is,

$$w_j \approx \frac{1}{n}\left.\frac{dQ}{dr}\right|_{r=j/n}. \tag{6}$$

We can now introduce an improved method to obtain the precise weights by applying equation (6) with the exact derivative and with j/n being replaced by $1\text{-}b_j$. The main reason of these choices is to make the DM satisfied by the evaluation measures of the criteria (which show how good they are) and not only by their orders. Thus we select the weights

$$w_j = \frac{1}{n}\left.\frac{dQ}{dr}\right|_{r=1-b_j}. \tag{7}$$

where $\{b_j\}$ is the ordered set of the inputs a_j with $b_1 \ge b_2 \ge \ldots \ge b_n$ or $(1-b_1) \le (1-b_2) \le \ldots \le (1-b_n)$. It can be easily shown from equation (7) that with a convex quantifier representing the pessimistic view, the w_j values will become larger for larger inputs rather than for smaller inputs. Reversely, the w_j values are larger for smaller inputs rather than for larger inputs by using a concave quantifier representing the optimistic nature of the DM. For example, the quantifier shown in Figure 3 is concave, which characterizes an optimistic DM. Its derivative is decreasing, therefore smaller value of $1\text{-}b_j$ generates larger weights. In the case of convex quantifiers, their derivatives are increasing, so larger value of $1\text{-}b_j$ generates larger weights.

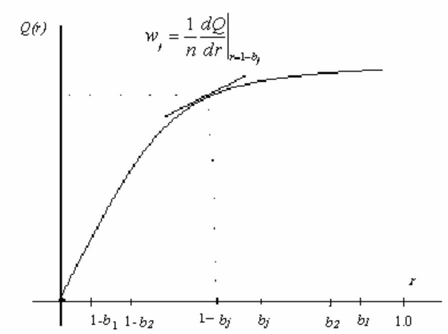

Fig. 3. Determining the OWA weights by using the derivatives of the RIM quantifiers

This method of weights selection can be called Precise OWA or POWA since it is based on the exact derivative of the quantifier. The POWA operator with weights (7) and any fuzzy linguistic quantifier is a neat operator since the combined measure, F_{POWA}, is independent of the ordering of the inputs:

$$F_{POWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j = \frac{1}{n}\sum_{j=1}^{n} Q'(1-b_j)b_j = \frac{1}{n}\sum_{j=1}^{n} Q'(1-a_j)a_j. \tag{8}$$

An additional advantage of using neat OWA operators in comparison to the original version of OWA is due to the fact that in this case, more attention is given to the context of the problem (e.g. to the values b_j). It is however a disadvantage of POWA that the weights have to be calculated separately for each alternative.

In reality the decision problem faces uncertainty in both the evaluations of the alternatives and in the optimism degree of the DM. Torra and Godo (1997) have investigated the case when only the evaluations of the alternatives are uncertain. In this study, we will consider uncertain optimism degree, where $f(\theta)$ is its PDF. Then the expectation and variance of F_{POWA} can be obtained as follows:

$$E(F_{POWA}) = \frac{1}{n}\sum_{j=1}^{n}\int_{0}^{1} Q'(1-a_j)\,a_j f(\theta)d\theta \tag{9}$$

and

$$Var\ (F_{POWA}) = \frac{1}{n^2}\int_{0}^{1}(\sum_{j=1}^{n} Q'(1-a_j)\,a_j)^2 f(\theta)d\theta - (E(F_{POWA}))^2 \tag{10}$$

These relations include fuzzy set theory since they are based on fuzzy linguistic quantifiers. They also include the stochastic density function to model the uncertainty of the optimism degree. Therefore they belong to the class of fuzzy-stochastic MCDM problems.

Special case

In this study RIM fuzzy linguistic quantifiers of the form $Q(r) = r^{\alpha}$ are defined in questioning the DM how many criteria he/she wants to consider. They are shown in Table 2. The DM is considered to be more pessimistic by evaluating the projects with respect to the more criteria. The optimism degree of the DM, θ, can be calculated by using fuzzy linguistic quantifiers. Yager (1996) derived the following relation for RIM quantifiers:

$$\theta = \int_{0}^{1} Q(r)dr = \int_{0}^{1} r^{\alpha}dr = \frac{1}{\alpha+1} \tag{11}$$

which can be obtained from equations (3) and (4) as n tends to infinity. Then $\alpha = \frac{1-\theta}{\theta}$, so the quantifier, Q, depends directly on θ.

Table 2- Particular RIM quantifiers, $Q(r) = r^{\alpha}$

Quantifier	Index of quantifier, α	Optimism degree, θ
At least one of them	$\alpha \to 0.0$, (α=0.01)	0.99
Few of them	0.1	0.9
Some of them	0.5	0.7
Half of them	1.0	0.5
Many of them	2.0	0.3
Most of them	10.0	0.1
All of them	$\alpha \to \infty$, (α=100)	0.01

The expectation of the combined goodness measures for an alternative is now calculated by using the special form of Q and FSPOWA (equation (9)) as follow:

$$E(F_{POWA}) = \frac{1}{n}\sum_{j=1}^{n}\int_{0}^{1} \alpha(1-a_j)^{\alpha-1}\,a_j f(\theta)d\theta. \tag{12}$$

that is

$$E(F_{POWA}) = \frac{1}{n}\sum_{j=1}^{n}\int_{0}^{1} (\frac{1}{\theta}-1)(1-a_j)^{\frac{1}{\theta}-2}\,a_j f(\theta)d\theta. \tag{13}$$

In this study we assume that the density function, *f(θ),* is defined by the triangular shape as shown in Figure 4. The value of *c* is the most likely value of the optimism degree.

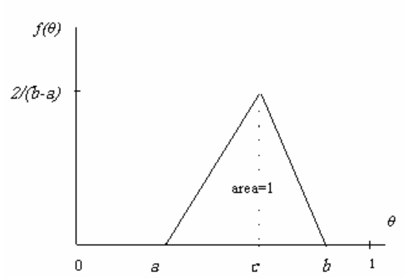

Fig. 4. A triangular density function *(a, c, b)*

The density function can be given as

$$f(\theta)=\begin{cases}\dfrac{2}{(b-a)(c-a)}\theta-\dfrac{2a}{(b-a)(c-a)} & if \quad a\le\theta\le c,\\[2ex] \dfrac{-2}{(b-a)(b-c)}\theta+\dfrac{2b}{(b-a)(b-c)} & if \quad c<\theta\le b.\end{cases}\tag{14}$$

As a further special case, we can reduce the number of model parameters**.** When $c<0.5$, that is the DM is pessimistic, the basin [*a, b*] of the stochastic optimism degree is selected as the interval [0, 0.5] and the most likely value, *c*, is on the top of the triangular density function. If the DM is neutral, ($c=0.5$), then the basin of the density function is [0.25, 0.75], and if he/she is optimistic ($c>0.5$), then the basin is [0.5, 1.0]. That is,

$$Basin \;\; of \;\; f(\theta)=\begin{cases}[0,\;\; 0.5] & if \quad 0\le c<0.5,\\[2ex] [0.25,\;\; 0.75] & if \quad c=0.5,\\[2ex] [0.5,\;\; 1] & if \quad 0.5<c\le 1.\end{cases}\tag{15}$$

The integrals in equation (13) have very complicated analytic expressions. Their values however can be obtained by applying Monte-Carlo simulation or routine numerical integrator (such as the Simpson's rule or Gauss-integrals). Each term of (13) has only two parameter a_j and *c*.

The variance of the combined goodness measure has a special meaning. The greater the variance, the larger is the uncertainty in the combined goodness measure for the various stakeholders, leading to lower level of the consensus. The level of consensus has therefore a direct relation with the robustness of the decision. The variance can be given as follows:

$$Var\;(F_{POWA})=\frac{1}{n^2}\int_0^1(\frac{1}{\theta}-1)^2(\sum_{j=1}^{n}(1-a_j)^{(\frac{1}{\theta}-2)}\,a_j)^2 f(\theta)d\theta-(E(F_{POWA}))^2\tag{16}$$

The first integral has two types of terms. The squares of each term need the computation of integrals of the form

$$\int_0^1(\frac{1}{\theta}-1)^2(1-a_j)^{(\frac{2}{\theta}-4)}\,a_j^2 f(\theta)d\theta.\tag{17}$$

These integrals also depend on two parameters, a_j and *c*. The cross-product terms depend on these parameters a_i, a_j and *c*:

$$\int_0^1 2(\frac{1}{\theta}-1)^2 a_i\,a_j(1-a_i)^{(\frac{1}{\theta}-2)}\,(1-a_j)^{(\frac{1}{\theta}-2)}\,f(\theta)d\theta.\tag{18}$$

Their values can also be computed by simulation or by using numerical integration, and the results can be tabulated by separate Tables for variables (a_i, a_j) with different values of *c*.

Case Study

There are thirteen water resources projects under construction in the Sefidrud watershed in the Northwestern region of Iran. These projects are concerned with reservoirs and their water distribution networks. The location of the watershed in the map of Iran and the schematic view of the dams are shown in Figure 5.

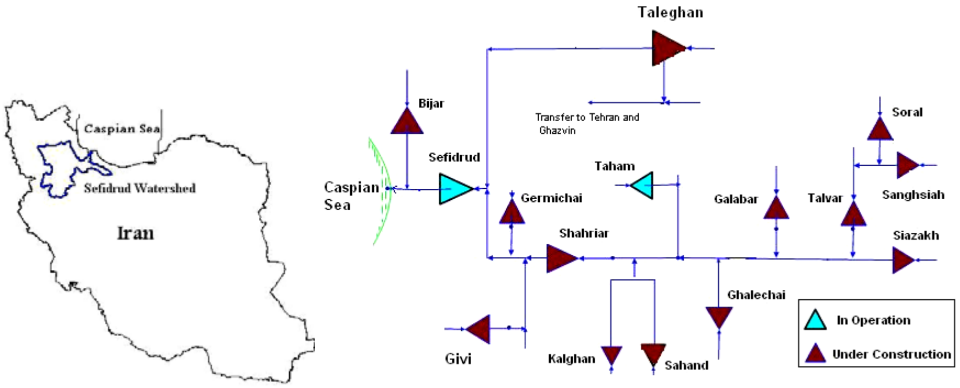

Fig. 5. Sefidrud watershed in the map of Iran and the schematic view of the dams

The DM in the watershed governing board has requested to find the most robust alternative among these projects with respect to seven criteria. The criteria and the evaluations of the projects with respect to these criteria were done by a group of experts. Their opinion is presented in Table 4.

Table 4- Evaluations of the water resources projects with respect to seven criteria

		Allocation of water to prior usages	Number of beneficiaries	Supporting other projects	Benefit/ Cost	Range of environmental impacts	Public participation	Job creation
		Weights of criteria						
Projects		*VH*	*SH*	*H*	*SH*	*H (negative)*	*H*	*H*
1	Sahand	M	VL	SH	1.07	VL	H	VH
2	Shahriar	SL	VH	SH	1.3	VL	SH	VH
3	Ghalechai	H	SL	L	1.1	L	VL	M
4	Kalghan	SL	L	M	1.2	L	SH	M
5	Germichai	H	M	H	1.02	L	M	H
6	Givi	SL	VL	VL	1.08	VL	H	H
7	Taleghan	SH	VH	VH	1	SL	SL	SH
8	Talvar	SH	VH	VL	1.2	VL	VL	SH
9	Galabar	SH	M	VL	1.43	VL	VL	VH
10	Sanghsiah	SL	L	SH	1	VL	VL	VH
11	Soral	SL	L	M	1	VL	VL	VH
12	Siazakh	SL	VH	H	1	VL	VL	VH
13	Bijar	H	VH	VH	0.8	L	VL	VH

Before applying the method, the original data of Table 4 are synthesized as follow:

Step 1. The evaluations of the projects with respect to the criteria are either linguistic or crisp numbers. The linguistic data are transformed into crisp numbers according to the scale shown in Table 5.

Table 5 - Linguistic variables and equivalent crisp numbers

Linguistic Variables	Number
Very Low	0.05
Low	0.20
Slightly Low	0.35
Medium	0.50
Slightly High	0.65
High	0.80
Very High	0.95

Step 2. The evaluations of the alternatives with respect to the criteria are normalized into the unit interval [0, 1] by using equation

$$a_j = \frac{f_j - f_{wj}}{f_j^* - f_{wj}}. \tag{19}$$

Here f_j is the evaluation of an alternative with respect to criterion j, f_j^* is the desired value of this criterion and f_{wj} is its worst value. For positive criteria they are selected as the maximum and the minimum values of the inputs, respectively. For negative criteria, they are selected in the opposite order.

Step 3. In the original version of OWA, the criteria weights are considered to be equal. However in this case the weights are different. Therefore the normalized evaluations of the alternatives are multiplied by their weights.

The expectations and variances of the combined goodness measures of the alternatives can now be computed by using equations (13) and (16). Since the DM selected the quantifier of '*many of the criteria*' the most likely optimism degree is c=0.3 according to Table 2. The expected values and the variances for each alternative are listed in Table 7 for this most likely optimism degree. The maximum expected values and the minimum variances are also indicated in Table 7 with bold face numbers.

Table 7- Expected values and variances of the combined goodness measures

	c =0.3	
Projects	$E(F_{POWA})$	$Var(F_{POWA})$
Sahand	0.2437	0.0154
Shahriar	0.2385	0.0270
Ghalechai	0.2100	**0.0023**
Kalghan	0.2407	0.0065
Germichai	**0.3117**	0.0163
Givi	0.1600	0.0074
Taleghan	0.2631	0.0096
Talvar	0.2125	0.0112
Galabar	0.1965	0.0157
Sanghsiah	0.1909	0.0050
Soral	0.1959	0.0039
Siazakh	0.1840	0.0129
Bijar	0.1647	0.0153

If the DM does not care about the risk in decision, his/her objective is to maximize the expected value of the combined goodness measure. If the DM cares only about the risk, then minimizing the variance of the combined goodness measure is his/her objective. According to the results with respect to the expected values, Germichai project is the most preferred project. Comparing the variances and knowing that smaller variance represents safer decision, it is clear that with respect to variance, this project is the second least preferred alternative. Therefore maximizing expectation and minimizing variances are conflicting objectives. Because of this conflict, we have to find a tradeoff between these objectives. Therefore we introduce a combination of expectation and variance, as it is usual in mathematical economics (Markowitz, 1959; Milgrom and Roberts 1992; Ballestero and Romero, 1996):

$$E(F_{POWA}) - \beta Var(F_{POWA}), \tag{20}$$

where β is a positive weight showing the importance of decreasing risk in comparison to increasing the expected payoff. The ranking of the alternatives by using the composite measure (20) with five various β values were computed, and the results are presented in Table 8.

Table 8- Ranking of the alternatives with various importance factors of the variance (*c*=0.3)

$\beta = 0.0$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 5$
Germichai	Germichai	Germichai	Germichai	Germichai
Taleghan	Taleghan	Taleghan	Taleghan	Taleghan
Sahand	Kalghan	Kalghan	Kalghan	Kalghan
Kalghan	Sahand	Sahand	Ghalechai	Ghalechai
Shahriar	Shahriar	Ghalechai	Sahand	Soral
Talvar	Ghalechai	Talvar	Soral	Sahand
Ghalechai	Talvar	Soral	Talvar	Sanghsiah
Galabar	Soral	Shahriar	Sanghsiah	Talvar
Soral	Sanghsiah	Sanghsiah	Shahriar	Givi
Sanghsiah	Galabar	Galabar	Galabar	Siazakh
Siazakh	Siazakh	Siazakh	Siazakh	Galabar
Bijar	Givi	Givi	Givi	Shahriar
Givi	Bijar	Bijar	Bijar	Bijar

Table 8 shows that Germichai is the most preferred and the most robust decision. Taleghan and Kalghan are the second and third most preferred projects while Givi and Bijar are the least preferred projects. Giving more importance to the variance (that is, selecting larger values of β) did not result in significant change in the ranking of the projects. The most probable value of β can be chosen by using repeated interactive process with the DM based on the outputs of the model.

Conclusion

Very few works have been done on fuzzy-stochastic MCDM models, especially with problems by discrete alternatives. In this paper a new method was introduced to take the uncertainty of the optimism degree into account by using fuzzy linguistic quantifiers and stochastic density functions. The original version of OWA operator was also revised. The new approach uses the criteria values to achieve a better characterization of the DM's satisfaction. Therefore it is a context based model in which the ordering of the initial inputs is not required, therefore it is a neat operator.

We considered the case of uncertain optimism degree, and in an important special case, we determined the expectation and variance of the combined goodness measure of the alternatives. The expectation was computed and tabulated for various parameter values. In order to reach reliable decisions a composite goodness measure was introduced that is a combination of the expectation and variance. This measure was used successfully in ranking the competitive alternatives in a case study, in which there are various stakeholders with different optimism degrees, therefore the application of this new measure can result in more satisfying decision and better consensus among the stakeholders.

References

Ballestero B, Romero C. (1996). Portfolio selection: a compromise programming solution. *The Journal of the Operational Research Society*, 47(11); 1377-1386.

Bath SK, Dhillon JS, Kothari DP. (2004). Fuzzy satisfying stochastic multi-objective generation scheduling by weightage pattern search methods. *Electric Power Systems Research*, 69; 311–320.

Bellman RA, Zadeh LA. (1970). Decision making in fuzzy environment. *Management Science*, Ser., B, 17; 141–164.

Ben Abdelaziz F, Lang P, Nadeau R. (1999). Dominance and efficiency in multicriteria decision under uncertainty. *Theory and Decision*, 47; 191–211.

Ben Abdelaziz F, Enneifar L, Martel JM. A multiobjective fuzzy stochastic program for water resources optimization: The case of lake management. *Information System and Operational Research Journal* 2004; 42(3); 201-215.

Bender MJ, Simonovic SP. (2000). A fuzzy compromise approach to water resource systems planning under uncertainty. *Fuzzy Sets and Systems*, 115(1); 35–44.

Buckly JJ. (1990). Stochastic versus possibilistic programming. *Fuzzy Sets and Syste*ms, 34; 173-177.

Caballero R, Cerda E, Munoz MM, Rey L. (2004). Stochastic approach versus multiobjective approach for obtaining efficient solutions in stochastic multiobjective programming problems. *European Journal of Operational Research*, 158; 633–648.

Changchit C, Terrell MP. (1993). A multi-objective reservoir operation model with stochastic inflows. *Computers and Industrial Engineering*, 24 (2); 303–313.

Chen CWE. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*, 114; 1-9.

Chen SJ, Hwang CL. (1992). *Fuzzy multiple attribute decision making*, Springer-Verlag: Berlin.

Das K, Roy TK, Maiti M. (2004). Multi-item stochastic and fuzzy-stochastic inventory models under two restrictions. *Computers & Operations Research*, 31; 1793–1806.

Eastman JR, Kyem PAK, Toledano J, Jin W. (1993). GIS and Decision Making. *Explorations in Geographic Information System Technology*, vol. 4, UNITAR: Geneva.

Hahn ED. (2006). Link function selection in stochastic multicriteria decision making models. *European Journal of Operational Research*, 172; 86–100.

Goicoechea A, Hansen DR, Duckstein L. (1982). *Multiobjective decision analysis with engineering and business applications*. Wiley: New York.

Inuiguichi M, Ramik J. (2000). Possibilistic linear programming: A brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem. *Fuzzy Sets and Systems*, 111; 3-28.

Iskander MG. (2005). A suggested approach for possibility and necessity dominance indices in stochastic fuzzy linear programming. *Applied Mathematics Letters,* 18; 395–399.

Lahdelma R, Salminen P. (2006). Stochastic multicriteria acceptability analysis using the data envelopment model. *European Journal of Operational Research*, 170; 241–252.

Liu L, Huang GH, Liu Y, Fuller GA, Zeng GM. (2003). A fuzzy-stochastic robust programming model for regional air quality management under uncertainty. *Engineering Optimization*, 35(2); 177–199.

Malczewski J. (2006). Ordered weighted averaging with fuzzy quantifiers: GIS-based multicriteria evaluation for land-use suitability analysis. *International Journal of Applied Earth Observation and Geoinformation*, 8; 270–277.

Maqsood I, Huang GH, Yeomans JS. (2005). An interval-parameter fuzzy two-stage stochastic program for water resources management under uncertainty. *European Journal of Operational Research*, 167; 208–225.

Markowitz H. (1959). Portfolio selection. Wiley: New York.

Milgrom J, Roberts P. (1992). *Economics, Organization and Management*, Prentice Hall: Englewood cliffs, New Jersey.

Mohammed W. (2000). Chance constrained fuzzy goal programming with right-hand side uniform random variable coefficients. *Fuzzy Sets and Systems*, 109; 107-110.

Mohan C, Nguyen HT. (2001). An interactive satisficing method for solving multiobjective mixed fuzzy-stochastic programming problems. *Fuzzy Sets and Systems*,117; 61-79.

Raju KS, Kumar DN. (1998). Application of multiobjective fuzzy and stochastic linear programming to Sri Ram Sagar irrigation planning project of Andhra Pradesh. *Proc. of 22nd National Systems Conference*: Calicut, India, 423-428.

Ribeiro RA. (1996). Fuzzy multiple attribute decision making: A review and new preference elicitation techniques. *Fuzzy Sets and Systems*, 78; 155-181.

Roubens M, Teghem J. (1991). Comparison of methodologies for fuzzy and stochastic multiobjective programming. Fuzzy Sets and Systems, 42; 119-132.

Sakawa M, Kato K, Katagiri H. (2004). An interactive fuzzy satisfying method for multiobjective linear programming problems with random variable coefficients through a probability Maximization model. *Fuzzy Sets and Systems*, 146; 205–220.

Shamsuzzaman M, Sharif Ullah AMM, Bohez ELJ. (2003). Appling linguistic criteria in FMS selection: Fuzzy-set-AHP approach. *Integrated Manufacturing Systems*, 14(3); 247-254.

Sinha SB, Hulsurkar S, Biswal MP. (2000). Fuzzy programming approach to multiobjective stochastic programming problems when *bi*′s follow joint normal distribution. *Fuzzy Sets and Systems*, 109; 91-96.

Slowinski R, Teghem J. (1990). A comparison study of STRANGE and FLIP, in: Slowinski R, Teghem J, (Eds), *Stochastic versus fuzzy approaches to multiobjective mathematical programming under uncertainty*. Dordrecht ; Boston: Kluwer, 392-393.

Stancu-Minasian IM. (1984). *Stochastic programming with multi objective functions*. Reidel: Dordrecht.

Teghem J, Dufrane D, Thauvoye M, Kunsch P. (1986). STRANGE: An interactive method for multi-objective linear programming under uncertainty. *European Journal of Operational Research*, 26; 65-82.

Torra V, Godo L. (1997). Averaging continuous distributions with the WOWA operator. *Proc. EFDAN' 97*, Dortmund: Germany.

Triantaphyllou E, Lin CT. (1994). Development and evaluation of five fuzzy multiattribute decision-making methods. *International Journal of Approximate Reasoning*, 14; 281-310.

Wang HF. (2000). Fuzzy multicriteria decision making-an overview. *Journal of Intelligent and Fuzzy Systems* , 9(1-2); 61 – 83.

Xu Z, Da QL. (2003). An overview of operators for aggregating information. *International Journal of Intelligent Systems*, 18; 953–969.

Xu Z. (2005). An overview of methods for determining OWA weights. *International Journal of Intelligent Systems*, 20; 843–865.

Xu Z, Chen J. (2007). An interactive method for fuzzy multiple attribute group decision making. *Information Sciences*, 177; 248–263.

Yager RR. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man and Cybernetics,* 18(1); 183-190.

Yager RR. (1993). Families of OWA operators. Fuzzy Sets and Systems, 59; 125–143.

Yager RR. (1996). Quantifier guided aggregation using OWA operators. *International Journal of Intelligent Systems*, 11; 49–73.

Yager RR. (2002). On the cardinality and attituditional charactersitics of fuzzy measures. *International Journal of General Systems*, 31(3); 303-329.

Zadeh LA. (1983). A computational approach to fuzzy quantifiers in natural languages. *Computers and Mathematics with Application*, 9; 149–184.

Zimmermann HJ. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1; 45-55.