



# Detection of Internal Erosion in Embankment Dams Using Temperature Measurements

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## Abstract

Temperature measurements represent methods for seepage monitoring of embankment dams. These procedures are able to detect the effects caused by time dependent processes such as internal erosion. The temperature in an embankment dam depends mainly on the temperature in the air and the water temperature in the upstream reservoir which vary seasonally. By temperature measurements, anomalous seepage areas could be located and the seepage flow could be estimated only after some days investigation. This paper presents analyses of monthly temperature measurements as input data of some case studies for seepage evaluations and internal erosion detections.

**Keywords:** embankment dam, internal erosion, seepage monitoring, temperature.

## Introduction

Most types of damage in embankment dams are related in some ways to internal erosion. The seepage flow increases slowly, almost coupled to the induced material transport that can take place over a long time. Therefore methods for seepage monitoring are required. Of particular importance are the methods which are able to register small changes in the seepage rate through a dam, and thus to detect internal erosion at an early stage before it starts to affect the safety of the dam. Experiences from all over the world indicate that the use of traditional methods of seepage monitoring is not perfect, and that a lot of dams need improved surveillance; moreover there are only a few monitoring methods that can be installed after the dam is constructed.

The temperature in an embankment dam depends mainly on the temperature in the air and the water temperature in the upstream reservoir. These two temperatures vary seasonally and create temperature waves propagating through the dam. Based on these assumptions, some models have been developed which describe the thermohydraulic process in a seepage zone. Using these models and by temperature measurements of embankment dams as input data, the seepage flow can be estimated and the areas with internal erosion can be located. Temperature measurements for seepage detection in dams started in Germany in the late 1950's (Kappelmayer 1957). Extensive research has also been performed specially in Germany and Sweden during the last 25 years (Armbruster 1983; Merkler et al. 1989; Johansson 1991 - 2007). By gaining acceptable and accurate results in seepage monitoring using the mentioned methods, now this technology is being recognized as the most effective way to detect seepage flow changes with high sensitivity all along the entire dam.

## Thermal Processes in Embankment Dams

The seepage monitoring method uses the seasonal temperature variations that occur in all surface water which causes a seasonal variation of the seepage water that passes through a dam. The magnitude of this seasonal temperature variation can be measured in the dam and is correlated to the seepage flow through the dam. Normally the seepage flow is small in embankment dams and the seasonal temperature variation in the dam depends essentially on the air temperature at the surface. The influence from the air is however less than 1°C for depths in the dam body that exceeds 10 m. At such depths the effect from the air is therefore negligible.

If larger quantities of water seep through the dam, the water temperature from the reservoir will affect the temperature inside the dam. At high seepage rates, the temperature variation of the water in the upstream

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reservoir completely determines the temperature inside the dam. The seasonal temperature variation in the dam is then directly proportional to the seepage rate.

The thermo hydraulic behavior of an embankment dam is complex. It includes such basic thermal processes as heat conduction (from the dam crest and from the foundation due to geothermal flow), advection and radiation. The first two processes are partly coupled to each other because viscosity and density of water are temperature dependent. The problem is further complicated by the variation in material properties in the dam, and the different conditions in the saturated and unsaturated parts of the dam. In order to analyze the problem, certain assumptions have generally to be made. The general problem can, however, be studied using coupled transport models. Both heat conduction from the surface, and geothermal heat can be ignored at many applications, which simplifies the evaluation.

Generally, a constant temperature will be a sign of a small seepage, while large seasonal variations may be a sign of significant seepage. At increasing seepage flow the temperature in the dam will be changed, and the seasonal variation will increase. This variation is dependent on seepage flow, the seasonal variation at the inflow boundary, and the distance from the boundary to the measuring point.

### Conduction in the Soil

When a temperature gradient exists in soil, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region. This kind of energy transferring is called conduction. Mainly the heat conduction in soil acts in vertical direction. This is due to the fact that, the geothermal flow acts upward from the base of the geotechnical structure and the temperature pulses caused by air temperature variations act downward. The geothermal flow for these calculations is small and therefore can be neglected. The one-dimensional heat-conduction Equation in soil can be obtained as follows. Using Fourier's law of heat conduction the following Equation can be written

$$q = -kA \frac{\partial T}{\partial x} \quad (1)$$

where  $q$  is the heat transfer rate (W),  $\partial T / \partial x$  is the temperature gradient in a meter length (m/K), and  $k$  is called the thermal conductivity of the soil (W/m.K). Considering the general case where the temperature may be changed with time and heat source may be presents within the soil, for the element of thickness  $dx$  the following energy balance may be made:

Energy conducted in left face + heat generated within element = change in internal energy + energy conducted out in right face.

Combining the related parts gives

$$-kA \frac{\partial T}{\partial x} + \phi A dx = r c A \frac{\partial T}{\partial t} dx - A \left[ k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dx \right] \quad (2)$$

or

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \phi = r c \frac{\partial T}{\partial t}$$

where  $T$  = temperature (K),  $x$  = coordinate (m),  $\phi$  = energy generated per unit volume (W/m<sup>3</sup>),  $c$  = specific heat of soil (J/kg .K) and  $r$  = density (kg/m<sup>3</sup>). This is one-dimensional heat conduction Equation. In three-dimensions the energy balance yields

$$q_x + q_y + q_z + q_{gen} = q_{x+dx} + q_{y+dy} + q_{z+dz} + \frac{dE}{dt} \quad (3)$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \phi = r c \frac{\partial T}{\partial t}$$

thus for steady-state one-dimensional heat flow in soil (no heat generation) the Equation (4) can be written

$$C_0 \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k_0 \frac{\partial T}{\partial x} \right) \quad (4)$$

where  $C_0$  = volumetric heat capacity of soil (J/m<sup>3</sup>.K). Using Equation (4) the one-dimensional heat-conduction temperature field of any geotechnical structure can be simulated.

### Advection and Dispersion Part of Energy Flux in Soil as a Porous Media

The conduction part of energy flux has been calculated in the previous section. The total energy flux in soil as a porous media contains heat advection caused by the leakage water flow. Moreover, some parts of the energy flux is due to mechanical and thermal dispersion because of the unresolved variability in the leakage water flow. Thus the total energy flux Equation can be written



$$C_0 \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k_0 \frac{\partial T}{\partial x} - C_w T q_i - Q^{disp} \right) \quad (5)$$

where  $C_w$  = volumetric heat capacity of water (J/m<sup>3</sup>.K),  $q_i$  = leakage flow (Darcy flow), (m/sec or m<sup>3</sup>/sec per m<sup>2</sup>) and  $Q^{disp}$  = energy flux due to mechanical and thermal dispersion (J/m<sup>2</sup>.sec). From the second term on the right hand side of the Eq. (5) a thermal velocity  $n_T$  can be defined

$$n_T = \frac{C_w}{C_0} q \quad (6)$$

The mass conservation Equation for a soil element with mass equal to  $m = r\Delta x\Delta y \times 1$  can be obtained

$$\frac{\partial M_{cv}}{\partial t} = \sum \dot{m}_{inlet} - \sum \dot{m}_{outlet} \quad (7)$$

$$\dot{m} = \frac{dm}{dt} = ru\Delta y + rv\Delta x \quad (8)$$

thus

$$\frac{\partial r}{\partial t} + \frac{\partial ru}{\partial x} + \frac{\partial rv}{\partial y} = 0 \quad (9)$$

where  $u$  = the velocity in x direction and  $v$  = the velocity in y direction. Therefore for one-dimensional water seepage in soil, with porosity  $n$ , the mass conservation Equation can be achieved.

$$\frac{\partial (r_f n)}{\partial t} + \frac{\partial}{\partial x} (r_f q) = 0 \quad (10)$$

where  $r_f$  = fluid density (kg/m<sup>3</sup>). The seepage velocity can be written in general form of Darcy's law

$$q = -\frac{k}{m} \left[ \frac{\partial P}{\partial x} + r_f g \right] \quad (11)$$

where  $m$  = dynamic viscosity (kg/m.sec),  $P$  = pressure (N/m<sup>2</sup>) and  $g$  = gravity (m/sec<sup>2</sup>). By inserting Equation (11) in Equation (10) the following Equation can be achieved

$$\frac{\partial}{\partial x} \left[ k \frac{\partial P}{\partial x} + r_f kg \right] = 0 \quad (12)$$

Equations (5) and (12) can be used in energy flux modeling of the soil. Using specified initial and boundary conditions of the problem as well as coupled Equations (5) and (12) the general form of heat and water flow in a porous media can be modeled.

## Thermal Models for Seepage Estimation and Internal Erosion Localization

Seepage evaluation and internal erosion localization of aquifers and embankment dams are the main aim of thermal models. By seepage monitoring of an embankment dam utilizing thermal methods, even in the first days, the development of internal erosion can be detected. Numerical modeling and phase delay analysis of a temperature pulse are the usual methods of seepage evaluation. The mentioned methods are used to seepage estimation at the first years of the development of thermal models. Later, using amplitude analysis some other useful mathematical methods are introduced. These models are discussed in the following sections.

### Numerical modeling of the temperature field

There are some numerical programs for heat and water flow modeling of a porous media. The most common program for energy flux modeling of an embankment dam is SUTRA, Saturated-Unsaturated TRANsport. The model may be employed for areal and cross-sectional modeling of saturated ground-water flow systems and for cross-sectional modeling of unsaturated zone flow. SUTRA energy-transport simulation may be used to model thermal regimes in aquifers, subsurface heat conduction, aquifer thermal energy storage systems, geothermal reservoirs, thermal pollution of aquifers, and natural hydrogeologic convection systems. The model employs a two-dimensional hybrid finite-element and integrated-finite-difference method to approximate the governing Equations (5) and (12).

Figure 1 depicts a SUTRA FEM model of a symmetrical embankment dam with a central clay core. The energy flux has been modeled for different permeability coefficients of the core. In addition, these analyses have been performed for embankment dams with various heights but the same thermal and geotechnical properties.

Normally the SUTRA simulations have been accomplished in three steps:



1. Calculation of the pressure distribution by iteration.
2. Obtaining initial conditions of problem using temperature variation timesteps of about one week for about some years temperature monitoring.
3. Simulating the last year with 1-3 days timesteps to achieve a high accuracy.

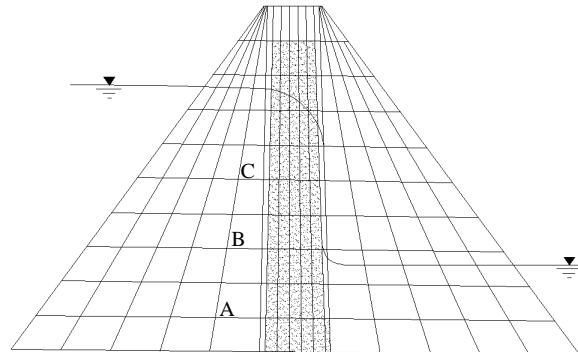


Figure 1- An embankment dam, modeled by SUTRA

By introducing initial and boundary conditions of the problem, SUTRA uses Equations (5) and (12) to approximate each node's temperature field. The temperature field of each node depends on the dam size and the amount of seepage flow through the dam. Figure 2a. shows yearly temperature variations of the point (B) in the Figure 1. Also, in Figure 2b. an arbitrary temperature pulse in the mid elevation of a dam with the height of 20 m can be observed. It can be seen that, depend on the dam size, both warm and cold temperature pulses may exist in large dams; however, in smaller dams because the pulse will pass through the dam within shorter time than the duration of the pulse, there is only cold or warm pulse. The minimum and maximum pulse temperatures are 6 and 13 °C respectively and thus there is almost a warm pulse in the dam.

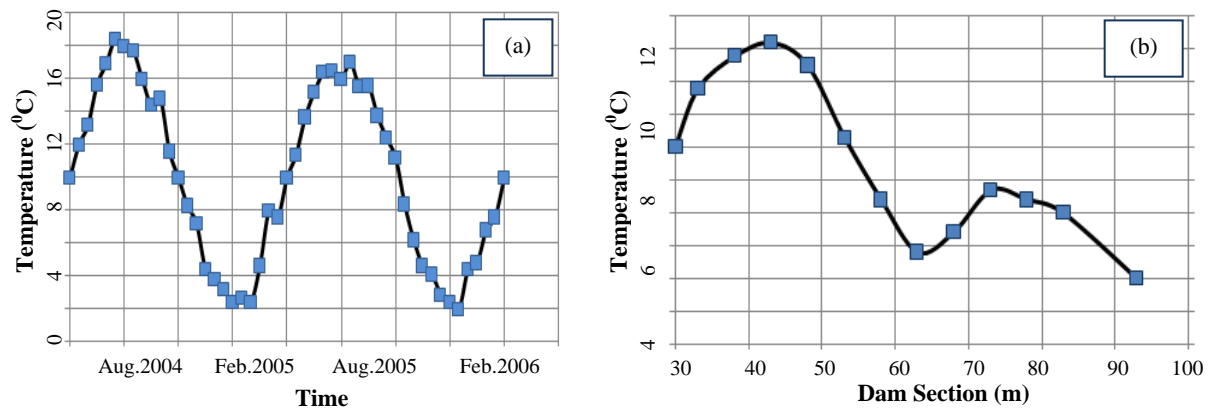


Figure 2- (a) Yearly temperature variations of the point (B). (b) The temperature pulse at the mid elevation of an embankment dam with the height of 20 m.

The model in Figure 1 is comparable with the Näs power plant in Sweden. Also Johansson (1991) studied the above temperature field model of the dam for obtaining the temperature field of the Näs power plant. By varying the permeability of the core and with the constant permeability of the earthfill ( $K=10^{-2}$  m/sec), temperature field of the nodes (A), (B), (C) (Figure 1) was investigated. The porosity of the core and earthfill was assumed to be 0.2. Comparing the maximum temperature differences ( $T_{\max} - T_{\min}$ ) with the permeability of the core, the simulations can be summarized (Figure 3a). It is obvious that, when the seepage flow is small ( $2-6 \times 10^{-7}$  m/sec) the temperature variations is also small, i.e. the predominant temperature process is conduction. In this leakage range the temperature pulses are almost constant. By increasing the seepage flow, the temperature variations will increase, i.e. the predominant temperature process is advection. These simulations have been also performed for symmetrical embankment dams with different heights. Figure 3b depicts temperature difference ( $T_{\max} - T_{\min}$ ) in node (A) in the upstream part of the dam, as a function of permeability and dam height (H). It is worth noting that, due to scaling, the relative locations of nodes are the same for the simulated dams. However, the absolute distance of the same nodes to the surface of dam has been changed. It is shown that, by increasing



the height of the dam the temperature variations will be decreased. Moreover, the temperature differences at low permeabilities are depending on this distance.

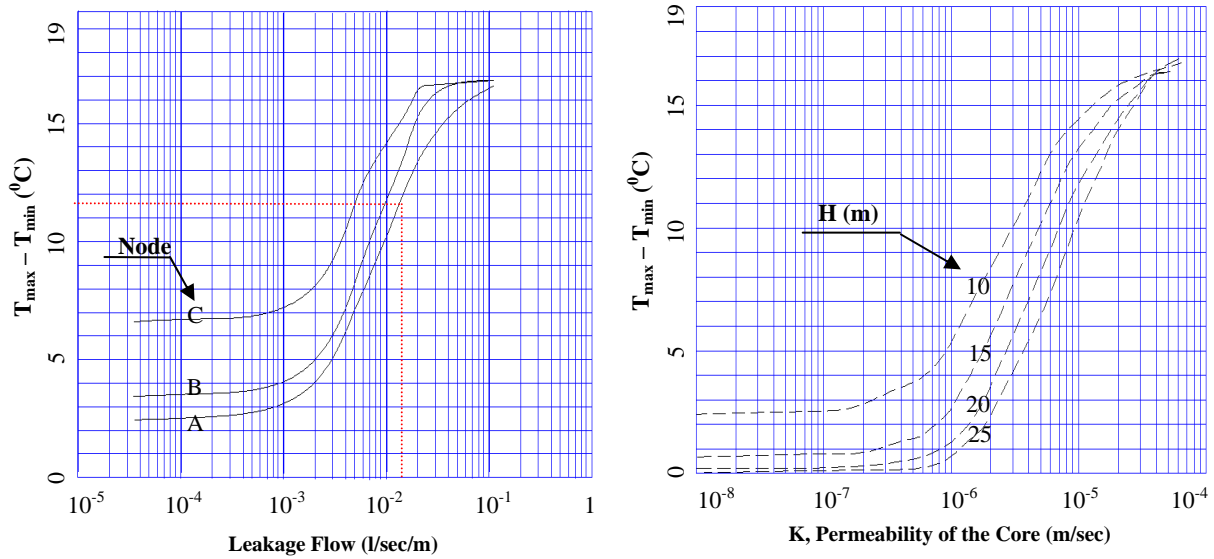


Figure 3- (a) The maximum temperature differences with the permeability of the core in a dam with a height of 10 m. (b) Temperature difference in node (A) as a function of permeability and dam height (H).

### Seepage Estimation of an Embankment Dam Using Numerical Method

To illustrate the utilization of the numerical method in seepage estimation and internal erosion localization of a real embankment dam, the Näs power plant of Sweden has been chosen. Näs dam has a height of about 10 m and crest length of 500 m. The dam is founded on bedrock close to the spillway and the power house. Other sections are founded on existing moraine layers. Näs dam is almost symmetrical with a central moraine core and is comparable with the SUTRA FEM model (Figure 1). The longest monthly temperature measurements at Näs power plant have been performed since 1987. However, after 13 years with regular temperature measurements, suddenly in the observation well V08, the constant seasonal variation substitute with the irregular variations. The

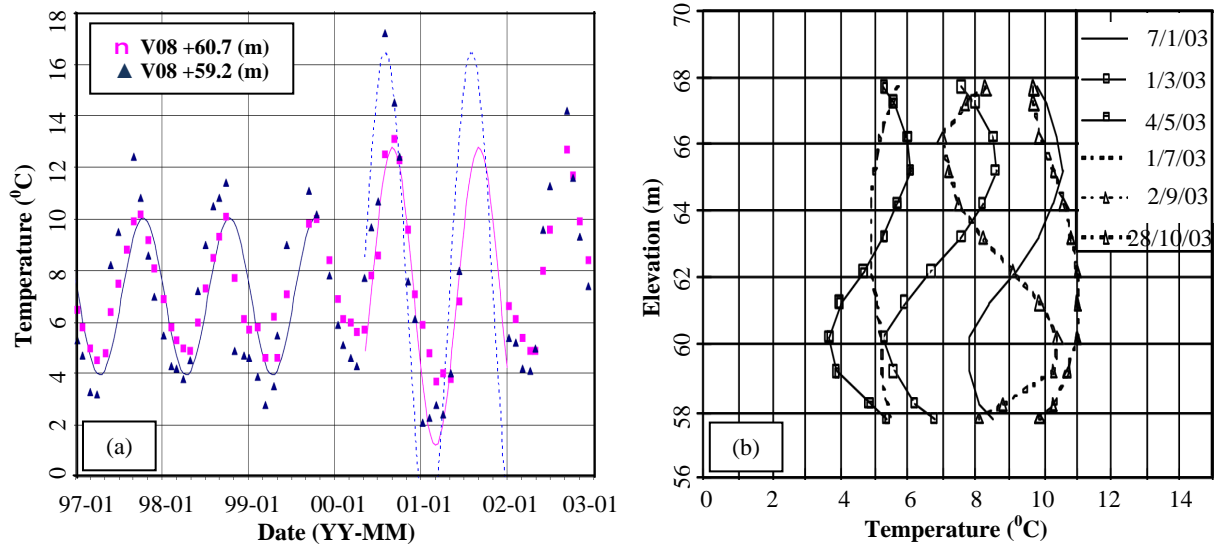


Figure 4- (a) Temperature at observation well V08 at two levels. (b) Temperature profiles in standpipe V08 in the embankment dam at Näs power plant

maximum temperature increased from 11 °C to 17 °C and the minimum temperature decreased from 4 °C to about 2 °C (Figure 4a). The only justification of these sudden variations was increasing seepage flow and thus beginning of internal erosion. A sinkhole was also observed close to this area in 2002. Figure 4b presents assumed vertical profiles of the mentioned well at different dates. As observed, there is an increasing seepage zone between elevation +59 m and +61 m. The height of the zone is about 2 m.



Node (A) in SUTRA model is almost situated at a level which approximately corresponds to elevation +59.2 m. From Figure 4a, it is obvious that the maximum temperature variations ( $T_{max} - T_{min}$ ) in the year 2000, for the observation well V08 at elevations +59.2 m and +60.7 m are 11.7°C and 17.2°C respectively. Using Figure 4a (the value is indicated by a dotted line) the leakage flow of  $q = 0.016$  (l/sec/m) can be specified to elevation +59.2 m. Due to the fact that, the internal erosion area's height from Figure 5b is almost 2 m, therefore the total seepage can be achieved

$$Q = q \times H \quad (13)$$

$Q = 0.016 \times 2 = 0.032$  l/sec.m or  $Q = 32 \times 10^{-6}$  m<sup>3</sup>/sec.m. Moreover, from Figure 3b the permeability of seepage zone has been increased from  $K = 0.79 \times 10^{-6}$  (m/sec) (in the previous year) to  $K = 3.2 \times 10^{-6}$  m/sec; in the other words, the permeability has increased about four times. Also in elevation +60.7 m the maximum temperature variations is about 17.3 °C. Because the value has no intersection with the dedicated curve of Figure 3a, thus the leakage flow is almost  $q = 1.0$  l/sec/m. Therefore by assuming the height of two meters for the seepage zone the total leakage is  $Q = 1 \times 2 = 2$  l/sec.m or  $Q = 2 \times 10^{-3}$  m<sup>3</sup>/sec.m. The seepage zone can be repaired by appropriate grouting. The grouting material due to the conditions of the dam may be sand-bentonite.

### Phase Delay Analysis

Phase delay analysis uses the time which a temperature pulse takes to propagate from the boundary to the measuring point. For a given distance between the measuring point and the boundary, the propagating velocity of temperature pulse can be obtained. By assuming that there is no heat conduction, which is a true assumption for large seepage flows or for large seepage zone thicknesses, the thermal velocity can be attained

$$v_T = x / t \quad (14)$$

By inserting Equation (14) into Equation (6), the leakage flow can be determined

$$q = \frac{C_0}{C_w} \times v_T \quad (15)$$

The seepage flow can now be calculated for known values of the volumetric heat capacity, which is assumed to be constant along the seepage path.

### Amplitude Analysis

The common case of internal erosion of embankment dams is increased seepage in a limited zone through the dam core. In the other words a permeable layer (seepage zone) is surrounded with two impermeable layers (core of the dam) which may have different properties. Usually, there is a concentrated constant horizontal seepage with the Darcy velocity  $q_w$  (m/s or m<sup>3</sup> water per m<sup>2</sup> and sec) in the homogeneous mid layer. The flow can be 10-1000 times higher in such damaged zones than in the undamaged part of the dam, where the seepage flow thus can be ignored; so there is pure heat conduction both in the upper and lower confining layers. The temperature field in the seepage zone is then mainly given by seepage flow (i.e. advection dominates) and the boundary condition. These assumptions are almost true for all of the embankment dams. Based on these assumptions the following mathematical model has been developed. By assuming a Decartian coordinates with an origin located at the left-below of the seepage zone (with the height of H), the temperature field of the model can be written

$$y > H: \quad \frac{\partial T}{\partial t} = \frac{k_1}{C_1} \times \frac{\partial^2 T}{\partial y^2} \quad (16)$$

$$0 < y < H: \quad \frac{\partial T}{\partial t} = \frac{k_0}{C_0} \times \frac{\partial^2 T}{\partial y^2} - v_T \frac{\partial T}{\partial x} \quad (17)$$

$$y < H: \quad \frac{\partial T}{\partial t} = \frac{k_2}{C_2} \times \frac{\partial^2 T}{\partial y^2} \quad (18)$$

where  $k_1, k_0, k_2$  are respectively the thermal conductivity of the upper, mid and lower layers. Also,  $C_1, C_0, C_2$  are respectively the volumetric heat capacity of soil in upper, mid and lower layers.  $v_T$  can be obtained using Equation (6). The boundary conditions of the problem can be given

$$y=H: \quad k_1 \frac{\partial T}{\partial y} \Big|_{y=H^+} = k_0 \frac{\partial T}{\partial y} \Big|_{y=H^-} \quad (19)$$

$$y=0: \quad k_2 \frac{\partial T}{\partial y} \Big|_{y=0^+} = k_0 \frac{\partial T}{\partial y} \Big|_{y=0^-} \quad (20)$$



The third boundary condition is based on the assumption that, the inlet temperature to embankment dam varies as  $\cos(\omega t)$  and  $\sin(\omega t)$  in time:

$$T(0, y, t) = T_0 \cos(\omega t) \quad (21)$$

where  $w = 2\pi/t_0$  and  $t_0$  is the time period. Using the boundary conditions of the problem, the problem will be same as a transient one-dimensional heat conduction problem in a slab. By solving the new problem using standard techniques such as Laplace transforms or separation of variables (Luikov 1968; Carslaw and Jaeger 1959) the desired temperature field will be attained. The following diagram is obtained using the model. In Figure 5  $T'$  and  $x'$  are respectively dimensionless temperature and distance which are defined as

$$T' = \frac{T_{\max,0} - T_{\min,0}}{T_{\max,1} - T_{\min,1}}, \quad x' = \frac{k_0 x}{C_w v_T H^2} \quad (22)$$

where  $T_{\max,1}$  and  $T_{\min,1}$  are the extreme values in yearly temperature cycle of reservoir and  $T_{\max,0}$  and  $T_{\min,0}$  are the mentioned values for the measuring point. By inserting Equation (6) in (21) the seepage flow can be estimated

$$q = \frac{k_0 x}{C_w x' H^2} \quad (23)$$

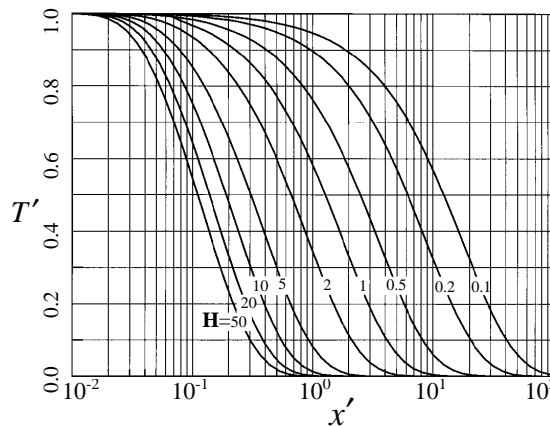


Figure 5- Relation between  $T'$  and  $x'$  for some vertical extensions  $H$  of the seepage zone.

### Seepage Estimation of an Embankment Dam Using Amplitude Analysis

Figure 6 presents temperature measurements of a 29 m height embankment dam. The dam geometry and geotechnical properties are similar to Ardak embankment dam located in north-east part of Iran. Figure 6a shows yearly temperature measurements of a standpipe V40 at different levels situated in the clay core. The distance between standpipe V40 from dam surface is 20 m. Moreover, the seasonal temperature variation of the Ardak River is shown in Figure 6a. As observed, the maximum temperature variation occurs at the lowest elevation (elevation +267 m) which is about one meter above the bedrock). At the mentioned level the atmosphere temperature effect is negligible and thus the temperature variation is due to the heat advection (because of the leakage flow). As can be seen, the maximum and minimum temperature values of elevation +267 (m) and reservoir are respectively 9.2, 1.6 and 15, 0.5 ( $^{\circ}\text{C}$ ). Thus the dimensionless temperature ( $T'$ ) can be obtained:

$$T' = \frac{T_{\max,0} - T_{\min,0}}{T_{\max,1} - T_{\min,1}} \quad (24)$$

Figure 6b shows the temperature profile of standpipe V40. It is obvious that there is a significant seepage zone in elevation +267 m. In addition, the height of the zone may be estimated about 1 (m). Thermal properties of the dam material and water are:  $k_0 = 2.30$  (W/m.K) and  $C_w = 4.18$  (MJ/m<sup>3</sup>.K). Using Figure 5 and with  $T' = 0.52$  and  $H = 1.0$  (m), the amount of  $x' = 0.98$  is obtained. The total seepage flow can be calculated from Equation (24).



$$Q = q \times H = \frac{k_0 x}{C_w x' H} = \frac{2.30 \times 20}{4.18 \times 10^6 \times 0.98 \times 1.0} = 11.23 \times 10^{-6} (m^3 / sec.m) \text{ or } Q = 11.23 (ml / sec.m).$$

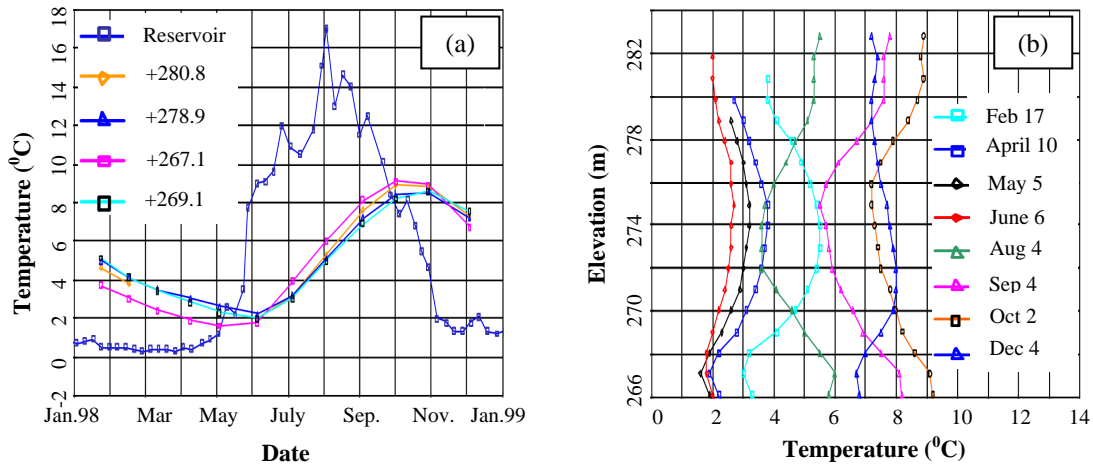


Figure 6- (a) Temperature in the reservoir and at selected elevations in 1998. (b) Temperature profiles in 1998.

### Concluding Remarks

Temperature method presents a useful seepage monitoring system for embankment dam surveillance. There are three thermal models for seepage flow estimation and internal erosion localization. The accuracy of the numerical method and phase delay method depend on the measurement technique adopted and can be further improved if calculations also are made for the actual dam. The amplitude method will be the best if there is an accurate estimation for the vertical extension of the leakage zone. However, in all of the models the natural variation of the thermal properties has a little influence on the required accuracy of the methods. At the last but not the least, by regular (once a month) and long term temperature measurements the interpretation of data will be easier and more accurate.

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