A FLOODING PREDICTION IN NATURAL RIVERS BY NUMERICAL CHARACTERISTICS SOLUTION

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Abstract

Flooding prediction in large rivers of compound sections is studied. The highly variable unsteady entry of water and sediment to a reach and changes in base level are amongst examples of natural events that induce changes in a river flow-sediment regime. Man-made troubles to nature such as construction of hydraulic structures and the implementation of river training techniques like bank stabilisation, channel realignment, dike construction, and dredging are some examples of artificial causes and problems imposed to river sediment transport regime. Generally a study of movable bed sediment propagation can be approached mathematically from either an analytical or a numerical stand-point. Analytical solutions can be obtained for simplified cases because the three governing equations (continuity and hydrodynamic of flow-sediment transport equations) describing unsteady open channel flow with a movable boundary are first order, non-linear partial differential equations of the hyperbolic type. Analytical models may be adequate only for an initial rapid evaluation of degradation and aggradation. Most situations, and in particular for highly variable flooding flows however, do not lend themselves to simplified solutions and more complete numerical methods then need to be adopted. In this paper numerical method of characteristics was used for prediction of flow-sediment prediction in large rivers.

Keywords: Flooding, Characteristics, Large Rivers, Flow-Sediment .

Flow-sediment equations for a flooding flow in natural rivers

A system of governing equations for alluvial channel flow can be derived by the application of the basic physical laws of conservation of momentum and conservation of mass (commonly known as continuity) to the water and sediment flow. Derivations of these equations for flow and sediment transport in movable bed channels can be seen from the references [1-3]. Different views based on various methods of flow-sediment transport available in literatures may be seen from Ref: [4-10]. For a flooding flow in system of movable bed channel the sediment mass inflow during an increment of time Δt, across the entire control volume is: $\rho_S \times (Q_S + q_{lS}\,\Delta x) \times \Delta t$. The sediment mass outflow is: $\rho_S \times [Q_S + (\partial Q_S/\partial x)\,\Delta x] \times \Delta t$. the change of storage of sediment mass is: $\rho_S \times [p_{or}\,(\partial A_d/\partial t) + \partial(AC_S)/\partial t\,] \times \Delta x\,\Delta t$, where ρ_S is density of sediment. Applying the definition of law of conservation of mass we can obtain:

$$\rho_s\left(Q_s + q_{ls}\Delta x\right)\Delta t - \left[Q_s + \left(\frac{\partial Q_s}{\partial x}\right)\Delta x\right]\times\Delta t = \rho_s\left[p_{or}\left(\frac{\partial A_d}{\partial t}\right) + \frac{\partial(AC_s)}{\partial t}\right]\times\Delta x\Delta t \qquad (1)$$

Ignoring sediment transport from flow-sediment transport together through a river, the following equation (2) is another form of continuity equation which is used in hydraulic modeling.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{\partial A_d}{\partial t} = q_l \qquad (2)$$

Hydrodynamic equation for flow-sediment transport in natural rivers when flooding occurs is defined as

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$$\rho\frac{\partial Q}{\partial t}+\beta\rho\frac{\partial}{\partial x}\left[\frac{Q^2}{A}\right]+\rho gA\frac{\partial y}{\partial x}-\rho gA(S-S_f)-\rho q_l\frac{Q}{A}+\rho\frac{Q}{A}\frac{\partial A}{\partial t}=0 \tag{3}$$

Characteristic methods of solution

The equations derived as above are the basis of the mathematical model used in this study. To solve these equations, they need to be reduced to three dependent variables, Q(x,t), A(x,t), and A_d(x,t), by the addition of ancillary relationships Equations (1-3) are non-linear differential equations. An analytic or closed solution of these equations is not available. Numerical Characteristic method of solution is used to solve the differential equations.

Flow-sediment characteristics

The characteristics method is widely used to solve the flow-sediment transport equations. In this method of solution the original system of partial differential equations is converted into the equivalent system of ordinary differential equations. At the first instance, equations 2 and 3 may be transformed to ordinary differential equations by a mathematical process as follows:

Ignoring lateral flow, equation 3 can be stated as

$$\frac{\partial Q}{\partial t}+2\frac{Q}{A}\frac{\partial Q}{\partial x}-\frac{Q^2}{A^2}\frac{\partial A}{\partial x}+g\frac{A}{T}\frac{\partial A}{\partial x}-gA(S-S_f)=0 \tag{4}$$

From combination of equation (2) and (4), another equation can be obtained as:

$$\frac{\partial Q}{\partial t}+2\frac{Q}{A}\frac{\partial Q}{\partial x}-\frac{Q^2}{A^2}\frac{\partial A}{\partial x}+g\frac{A}{T}\frac{\partial A}{\partial x}-gA(S-S_f)+\varepsilon\left[\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}\right]=0 \tag{5}$$

Where ε is constant. In equation 6, the first and second terms are total derivatives of Q and A respectively if: $dx/dt = 2Q/A + \varepsilon$ and, $dx/dt = 1/\varepsilon\,[gA/T - Q^2/A^2]$.

Equating equations (4) and (5) gives, $\varepsilon = - Q/A \pm [(gA/T)^{0.5}]$. Substituting these values of ε in $dx/dt = 1/\varepsilon\,[gA/T - Q^2/A^2]$, e q u a t i o n (5) leads to,

$$\frac{\partial Q}{\partial t}-\frac{\partial A}{\partial t}\left[\frac{Q}{A}+(g\frac{A}{T})^{05}\right]-gA(S-S_f)=0 \tag{6}$$

This is when dx/dt = Q/A + [g A / T]0.5 = U + c, which is positive characteristic.

$$\frac{\partial Q}{\partial t}-\frac{\partial A}{\partial t}\left[\frac{Q}{A}-(g\frac{A}{T})^{05}\right]-gA(S-S_f)=0 \tag{7}$$

And this is when q dx/dt = Q/A - [g A / T]1/2 = U - c which is negative characteristic. Where; c is the velocity with which a disturbance in open channel flow tends to move over the water surface, and is measured relative to the water (not to the banks).

Equations 6 and 7 are equivalent to the system of partial differential equations 2 and 3. These are two ordinary differential equations that apply along the curves defined by positive and negative characteristics respectively.

The problem of wave (water and sediment) propagation can be described by a system of hyperbolic partial differential equations:

$$\frac{\partial f}{\partial t}+A\frac{\partial f}{\partial x}+B=0 \tag{8}$$

Where, *A* and *B* are square matrices (3 × 3) of coupling coefficients of ∂f/∂t and ∂f/∂x, in which f represents Q, A and A_d terms in equations (1- 3).

The system of equation (8) is hyperbolic if the matrix A has 3 roots c_1, c_2, and c_3, associated with three linearly independent characteristic vectors. The characteristic roots can be obtained by imposing a zero value for the following determinant.

$| A(i,j)dx/dt - B(i,j) | = 0$

In order to study the set of governing equations, the derivative of the sediment discharge, Q_S, with respect to space and time can be expressed as a function of the dependent variables:

$\partial Q_S/\partial x = \partial Q_S/\partial Q\ \partial Q/\partial x + \partial Q_S/\partial A\ \partial A/\partial x$, $\partial Q_S/\partial t = \partial Q_S/\partial Q\ \partial Q/\partial t + \partial Q_S/\partial A\ \partial A/\partial t$,

$\partial C_S/\partial x = \partial/\partial x\ (Q_S/Q)$, and $\partial C_S/\partial t = \partial/\partial t\ (Q_S/Q)$.

After expressing total derivatives of the three dependant variables; *Q, A* and A_d in equations 1-3 with respect to time and space, the coefficient matrices *A(i,j)* and *B(i,j)* can be expressed in the form of; $\alpha c^3 + \beta c^2 + \gamma c + \delta = 0$
Where $\alpha = p_{or} - A/Q\ \partial Q_S/\partial A + Q_S/Q$, $\beta = gA^2/Q\ p_w (\partial Q_S/\partial Q - Q_S/Q) - 2\ pQ/A + A/Q\ (Q^2/A^2 - gA/T)(\partial Q_S/\partial Q - Q_S/Q) + \partial Q_S/\partial A + 2\ Q/A\ (A/Q\ \partial Q_S/\partial A + Q_S/Q)$,
$\gamma = gA / p_w (A/Q\ \partial Q_S/\partial A + Q_S/Q) + p_{or}(Q^2/A^2 - gA/T) - gA/p_w\ \partial Q_S/\partial Q - \partial Q_S/Q\ (Q^2/A^2 - gA/T) - 2\ Q/A\ \partial Q_S/\partial A$, and $\delta = - gA/p_w\ \partial Q_S/\partial A$

In the above equation three roots; c_1, c_2, c_3 has three characteristics in the plane (x,t). Each point in the solution domain, including the boundaries, is an intersection of the three characteristic curves whose directions correspond to the three roots of the system. In order to find these roots, the equation can be solved using Newton's approximation. Thus, assuming an initial value for the roots of the equation as:
$c'_1 = 0$ (for the bed wave), $c'_2 = Q/A + (gA/T)^{0.5}$, $c'_3 = Q/A - (gA/T)^{0.5}$ (for the hydraulic solution), where c'_1 is for the celerity of bottom perturbations and c'_2 , c'_3 are for the free-surface wave celerity. The c'_1 is smaller than the other ones, requires a second order approach to determine its value. Applying $c_1 = c'_1 - f(c'_1)/ f'(c'_1) = -\delta/\gamma$ and substituting values of δ and γ from the above and ignoring small terms, the wave velocity for the bed wave can be as: $c_1 = (gA/p_w\ \partial Q_S/\partial A) / p_{or}(Q^2/A^2 - gA/T)$ or, $c_1 = T / p_{or}\ p_w\ [1/(Fr)^2 - 1]\ \partial Q_S/\partial A$. Similarly, for the water wave: $c_2 = Q/A + (gA/T)^{0.5} + T / 2p\ p_w (1/1 + Fr)\ \partial Q/\partial A$, and $c_3 = Q/A - (gA/T)^{0.5} - T / 2p\ p_w (1/1 - Fr)\ \partial Q/\partial A$.

Derivations of the above terms are valid for the both sub-critical and super-critical flows. In supercritical flow the initial conditions are the same as sub-critical flow, while the hydraulic boundary conditions are to be imposed at the upstream and the sediment boundary condition must be imposed downstream (anti-dunes).

Ancillary relationships

For sediment routing, the equations (1-3) contain three basic unknowns Q, A and A_d. The other variables in the equations must be expressed as a function of these three unknowns, in order to obtain a solution. S_o, the variables other than Q, A and A_d which need to be defined are the hydraulic depth, the bed slope S of the channel, the frictional slope S_f and the lateral flow. Two extra quantities also need to be defined in sediment routing which are the sediment discharge, Q_S, and concentration, $C_S = Q_S/Q$.

Friction slope, S_f, and sediment discharge Q_S need to be determined by ancillary relationships. The other above mentioned variables can either be measured from the geometry of the river or they must be defined by additional ancillary equations. The geometric properties can be obtained from survey information on the river reach. Cross-sectional data will enable both cross-sectional area, A, and hydraulic radius, R, to be expressed in terms of depth y. Bed slope, S, can be obtained from long-section data.

The frictional slope, S_f which appears in the momentum equation may be related to the flow parameters, expressing it as a function of flow and channel characteristics.

The friction slope may be calculated as: $S_f = [\alpha\ Q / A\ R^{\beta}]^2$, where α and β are constants [$\alpha = n$ and $\beta = 2/3$]. The roughness n can also be defined in terms of the Darcy-Weisbach friction factor and the Chezy's C. It may be written as: $n = [(fR)^{1/3} / 8g]^{0.5} = R^{1/6} / C$.

Application and Discussion of Results quations

There are many practical applications of the characteristics method such as: prediction of flow-sediment variation upstream of a dam, prediction of flow-sediment in a dredged channel, and prediction of flow-sediment variations in a channel reach with tidal flow near an estuary. At least one example flow-sediment variation upstream of a dam was studied here.

Flow-sediment upstream of a dam

A river location upstream of a dam was selected because of important role of a dam in flooding detention. As the river approaches the dam, the cross-sectional area of the flow is increased, and the average velocity of flow is decreased which in turn causes a reduction in transporting capacity of the flow. The difference between transporting capacities causes sediment deposition and changing the water surface level, bed level and topography of the upstream and the lake behind the dam.

A test run of the flow and sediment discharge at the upstream was taken unsteady when flooding takes place. A wide rectangular channel was selected. The reach length was taken as 20 km with special increment of $\Delta x = 500$m and the bed slope $S = .0005$. An upstream variable flow hydrograph, Q_V, was introduced as; $Q_V = Q_b + (Q_p - Q_b)\ [t/t_p \exp(1 - t/t_p)]^{16}$ with a base flow, $Q_b = 5 m^3/s$ per unit width rising to a peak flow, $Q_p = 15\ m^3/s$ per unit width. The time to peak t_p was taken as 12 hours. The sediment load at the upstream boundary were defined by; $Qs_{in} = Qs_b + (Qs_p - Qs_b)\ \{t / t_p \exp [1- t / t_p]\}^{16}$. Where, the sediment base flow Qs_b was taken to be $0.00055 m^3/s$ per unit width and peak sediment flow Qs_p was set at 25 times the Qs_b value, i.e. $0.01375 m^3/s$ per unit width. A uniform sediment bed material was selected. The value of volume of sediment per unit volume of bed layer, p_{or}, was 0.8, and effective grain size (d_{35}) was taken as 0.8 mm. A simple equation: $Qs = \alpha_1/y\ (Q/A)^{\beta 1}\ Q$ was used for Qs_{in} calculation. Where; $\alpha_1 = 0.00008$ and $\beta_1 = 3$ values of constants α_1 and β_1 were estimated by comparison with the Ackers and White total load formula for d_{35}= 0.8 mm. The constant stage condition was used to define the downstream boundary condition.

Selection of an appropriate numerical flow-sediment routing model for a particular application depends upon the variability of flow, depth and sediment discharge as well as the influence of the boundary conditions.

The numerical testing of the linear/nonlinear, coupled/uncoupled solutions were studied earlier [3]. Comparison is not the case studied here. Concerning to different methodology and solutions available for prediction of flooding flow-sediment through natural rivers, the solution adopted here having continuous interaction of the hydraulic and sediment transport phases is taken into account. This clearly makes better physical sense for flooding in which flow and sediment phases are coupled together and solved in this study. The choice of a time increment in uncoupling flow phase from sediment solution may tend to be conservative, as it cannot be foreseen at the beginning of a time increment how large the computed flow-sediment changes will be. Certainly, the methodology employed here can generally be considered as the most suitable method for river flow-sediment routing applications, because it is coupled flow-sediment phases which can best represent the physical process, particularly applying to flooding which was subject of this work.

Conclusion

The result of application for the methodology used in this study is compared with the measured one as shown in Figure 1. The data used for comparison is taken from reference [3]. The result of this study shows that the characteristics method is an accurate method for studying many types of gradually varied unsteady flow-sediment problem. A fixed grid characteristics method by interpolation can be used with significant advantage over other finite difference scheme please see ref [3].

Finite difference explicit method of fixed grid schemes sometimes may be preferred because of their computational simplicity, but they are prone to instability from a numerical viewpoint unless a relatively small time step is used. The time step for the finite difference explicit scheme is limited by the Courant condition. Different methods have been compared for the solution of the Saint-Venant equations, and it is found that characteristics method is one of the most efficient solutions with no stability condition problems [3].

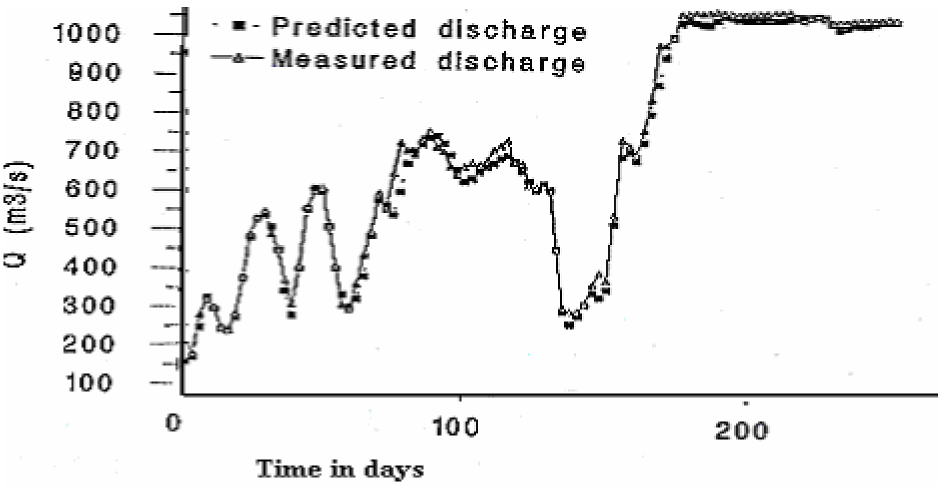

Figure 1– Measured and predicted discharge flooding prediction

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