SENSITIVITY OF SEDIMENT PREDICTION TO SIZE GRADATION

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Abstract

Varied unsteady flow-sediment equations for a graded river bed contain a variety of unknown sediment characteristics, hydraulic, and geometric parameters. These parameters, together with input data comprising initial and boundary conditions, would be required to simulate flow-sediment variations. Sensitivity analysis and optimisation procedure involves determining these parameters by fitting the model to either field or simulated data. Prior to optimisation a sensitivity analysis of existing flow-sediment parameters was found useful in highlighting the role of each parameter and reduced the time and number of iterations in the solution process. In this paper an optimization technique is employed for finding best fitted value of some important parameters involved in graded sediment routing and bed armouring processes using Advanced non-linear coupled model (ANCMG). The particular case study used for this investigation is degradation, bed armouring and grain size distribution of bed material in a river bed downstream a dam. For a graded river bed, the effects of bed roughness, sediment parameters and thickness of the active layer on sediment sizes, and bed level changes were studied. A combined Gauss-Newton and modified Newton method [9] was employed to calculate the optimised parameters.

Keywords: Graded sediments, NCMG model, sensitivity analysis, optimization

Introduction

The aim of the present paper is the development and objective calibration of one dimensional sediment routing numerical model to deal with river bed material of homogenous and mixed grain size characteristics.

In this study, conservation of mass and semi-empirical equations governing sediment particle movement are adopted to establish the interaction between the sediment movement and fluid flow. The resulting mathematical formulation is highly non-linear and complex. It is impractical, if not impossible, to solve it analytically. Therefore governing equations of sediment laden water continuity, sediment continuity, and momentum for flow-sediment are solved simultaneously all together numerically.

The three governing equations can be solved in an approximate linear form or in the more complete non-linear form. Also, by ignoring certain terms, the sediment continuity equation can be uncoupled from the other two. Algorithms have been developed for non-linear and fully coupled solutions. Solutions were obtained with the grain sorting / armoring included or excluded.

Briefly the objectives of the present paper are:

1) A non-linear fully coupled numerical model (NCM) is developed to determine the geometrical, sediment and hydraulic conditions in which it may be the preferred method compared its performance with the existing linear coupled model (LCM), linear uncoupled model (LUM), and non-linear uncoupled model (NUM) developed previously [5].

2) The non-linear coupled model extended to deal with graded sediments (NCMG).

3) A grain sorting/armoring algorithm is developed and incorporated with the NCMG model, tested with the published laboratory data, and employed through the optimization process

4) The behavior and stability of the NCMG model investigated through its sensitivity to changes in various model parameters through simulated solutions.
5) An optimization technique applied to identify friction and sediment transport parameters in the NCMG model. The model is successfully applied to observed data from the Fork, and Missouri rivers in the USA. A system of governing equations for flow-sediment transport of graded bed material through natural rivers was derived by application of the basic physical laws of conservation of momentum and conservation of mass to the water and sediment transport.

Unsteady varied flow-sediment parameters

Following unsteady varied flow-sediment relationships were considered;

Equation for each particle of a graded bed sediment material involved in active layer exchange:

$$\partial Qs/\partial x + p\partial Ad/\partial t + \partial A\ Cs/\partial t = q_{ls} \tag{1}$$

$$Qs = \alpha_1\ (Q/A)^{\beta 1}\ (Q/d_m)^{\gamma 1} \tag{2}$$

Continuity equation for transport of sediment-laden water:

$$\partial Q/\partial x + \partial A/\partial t + \partial Ad/\partial t = q_l \tag{3}$$

Dynamic equation for transport of sediment-laden water:

$$\rho\ \partial Q/\partial t + \beta\ \partial/\partial x\ [\rho\ Q^2/A + \rho g\ A/T\ dA/\partial x - \rho gA\ (S - S_f) - \rho\ q_l\ Q/A + \rho\ Q/A\ \partial Ad/\partial t = 0 \tag{4}$$

$$S_f = [n\ Q\ /\ A\ R^{2/3}]^2 \tag{5}$$

The different parameters used in above equations are: Q is the discharge; A is the area of cross-section; A_d is the volume of sediment deposited/eroded per unit length of channel; x is the distance along the channel; t is the time; q_l is the lateral flow per unit length of channel; β is the momentum correction factor; g is the acceleration due to gravity; T is the channel top width; S is the bed slope and S_f the friction slope; Q_s is the sediment discharge for each size fraction, C_s is the suspended sediment concentration, p, represents the volume of sediment in unit volume of bed layer and q_{ls} is the lateral sediment flow. The above set of equation requires two supplementary equations for their solution. Equations are required to relate frictional slope S_f and sediment discharge Q_s to hydraulic and geometrical variables.

Optimization of graded sediment routing

There are a number of formulae for predicting bed friction and sediment discharge, as well as a variety of grain sorting and bed armoring algorithms for non-uniform sediments. Most of them depend on empirical parameters. For example, the Manning's equation for calculation of friction slope contains a "roughness" or more correctly channel resistance parameter. It depends on many variables such as bed material and bed form, vegetation cover, surface roughness of the channel, channel geometry and alignment, silting and scouring obstructions, stage and discharge, and seasonal changes etc. In the case of natural rivers, because of uncertainty in some of the variables listed above, an a priori estimation of this parameter would be rather difficult. Optimization provides a means of doing this.

Bed materials sorting, exchange of sediment sizes, and armoring based on the complete unsteady varied flow-sediment equations, including optimizing the flow-sediment parameters in a movable graded bed river is studied.

The correct assessment of many factors and major parameters involved in the process including flow-sediment discharge, characteristics of sediment bed materials, hydraulic resistance, active layer thickness, and particles exchange were considered.

In this study optimization of important flow-sediment parameters followed by sensitivity analysis to simplify the mathematical complexity, reduce iteration process, and computing error. Equations for prediction of flow-sediment discharge, friction factor, the grain-sorting and armoring algorithm involve the minimum number of parameters while representing a real condition as closely as possible.

Among the different friction formulae widely used in research, a general form of Manning equation (equation **5**) is used for friction prediction. In this equation, the resistance parameter, n, is optimized.

It is a very difficult task to find a single sediment transport and bed armoring algorithm for predicting all natural conditions involved in a real river. There are many factors, which may influence the sediment discharge, grain sorting and bed armoring processes. These are; bed material composition of different grain sizes, transported material of different sizes which may vary with distance and with time and may be different from the distribution of the original bed material, stage and discharge transport dependent bed resistance, and the collapse of cross-section sides, etc. This is why none of the available formulae are totally comprehensive and applicable to all conditions with confidence. Based on these facts it may be more reasonable to use a method involving the major important parameters to model a given reach and to optimize its parameters. Thus a flow-sediment transport relationship (equation **2**) developed in this study to describe situations where parameters are adjusted, by optimization, to account for complex variations of flow-sediment and geometric conditions.

Predicting the sediment load, grain sorting and bed armoring of graded sediment, requires a sediment transport formula which takes the effect of different grain sizes into account. The major important sediment parameters in the Ackers-White formula and the developed equation are identified and optimized. These parameters contain properties of the hydraulics, channel geometry, sediment characteristics and bed material composition which exist at a particular location. The simple grain sorting and armoring algorithm [1] is used. The active layer in the armoring algorithm, which is effectively engaged in exchange with water is a very important parameter in a numerical model. Composition of the active layer thickness may vary with changes in hydraulic conditions, and the length of the computational time step, etc. Therefore the active layer thickness is also treated as an optimization parameter in the numerical model.

Optimization method

Gradient technique was considered to be the most suitable technique for objective functions based on the sum of squares of errors criterion. However since an analytical formula is not available for derivatives of the objective function in this case, they need to be calculated numerically if first or second order gradient techniques are to be used.

A NAG FORTRAN library subroutine named E04FDF [**9**], based on the least square technique was used here for the purpose of calculation. This NAG routine uses the Gauss-Newton and modified-Newton method. There is no need to provide derivatives of the objective function in this technique, since the subroutine itself calculates the derivatives numerically. Decisions about the values of the parameters for each iteration, if required, are made based on these numerical values of the derivatives.

The E04FDF routine is an easy-to-use algorithm for finding an unconstrained minimum of a sum of squares of **m** non-linear functions in **n** variables (**m** ≥ **n**). It is intended for functions which are continuous and which have continuous first and second derivatives, however it usually works even if the derivatives have occasional discontinuities. The routine is identical to the subroutine LSNDN1 in the National Physical Laboratory Algorithms Library. It is applicable to problems of the form

$$\text{Minimise } E = F(a) \qquad \textbf{(6)}$$

Where, $a = (a_1, a_2, a_3 \ldots, a_n)$, and $m \geq n$.

The functions $F(a)$, are referred to as residuals. A subroutine with the name of LSFUN1 must be written to evaluate functions $F(a)$, at any point. For the problem in hand $F(a)$ is described with respect to bed level changes as;

$$F(a) = E == \sum_{i=1,m} \{ y_{bo}(i) - y_{bs}(i) \}^2 \qquad \textbf{(7)}$$

Where, y_{bo} is the observed bed level which is known, and y_{bs} is the simulated bed level to be calculated and supplied by LSFUN1 to the E04FDF NAG Routine in the model. From a starting point supplied known as the initial guess, a sequence of points is generated which is intended to converge to a local minimum of the sum of squares. These points are generated using estimates of the curvature of $F(a)$.

Objective function and system parameters:

In equation (6), the function $F(a)$ is an objective function in which the value of E requires to be minimised. The variables (a) whose values are to be adjusted to obtain the minimum are called the system parameters. The objective function is considered as a measure of the difference between the required and the actual performance of the system. For the problem in hand, of the graded sediment routing problem, the model parameters are adjusted. These are; roughness parameter in the friction equation, sediment parameters in the sediment transport equation, and the thickness of active layer in the armoring algorithm need to be adjusted so that errors in the predicted bed level values are minimized. The model parameters are used as the system parameters and the resulting errors combined to form the objective function.

The objective function here is often considered as a measure of the difference between the required and the actual performance of the system. The optimization techniques applied here are employed for adjustment of bed level variations, grain sorting and bed armoring processes of a graded river bed. The main dependent variable of the objective function used here is bed level change. This is firstly chosen, since it is widely available in data collection. It is an essential parameter in the design of hydraulic structures. Moreover, all of the important sediment parameters, hydraulic parameters, bed roughness, and thickness of the active layer are involved in its calculation.

Numerical examples of optimization

The single and multi-variable optimization techniques are applied here to a selected case problem, erosion downstream of a dam. The application of the optimization methods are performed with a set of hypothetical data.[5]

The hypothetical data is used because, the problem can be better controlled and analyzed. The optimized values are known so a check on the effect of errors in the first estimate is simple, the rate of closure to the minimum error is easily monitored, and also a check on whether the solution is closing to a local or global minimum is possible. Experience gained from the hypothetical test will help to run optimization tests better for the real river situation to be looked at later.

Case study graded bed degradation downstream of a dam

The application of optimization to hypothetical data is performed here. The NCMG model is used for a graded sediment routing problem. The example, degradation of a graded bed downstream of a dam is used for testing the model.

Optimization of parameters using simplified equation

The optimization tests were carried out in two stages. First a solution of the problem with reference values of the parameters was considered as observed data. Then the reference values of the roughness parameter, α, in the friction equation, parameters, α_1, β_1, γ_1 in the sediment transport equation, and thickness of active layer, T_a, in the armoring algorithm were changed by set amounts. Optimization based on the gradient method {**NAG** subroutine **E04FDF**[9]} Gauss-Newton and modified Newton method was then applied to identify the true values of these parameters by iteration.

The criteria for the termination of the optimization process were set by the **NAG** routine as follows:

a) the allowable error in the objective function was reached;

b) there have been a maximum number of subroutine calls (iterations);

c) the objective function did not approach to a minimum, but no lower point could be found.

The problem was solved initially to obtain the observed data set. The values of the parameters, $\alpha = 0.036$, $\alpha 1 = 0.01$, $\beta 1 = 4.$, $\gamma_1 = 0.1.$, and $T_a = 0.006$ **m** were each then adjusted and optimized using the observed data set.

The objective function, based on the sum of the squares of the errors, was adopted to optimize parameters; α, α_1, β_1, γ_1 and T_a, for the bed level changes along the channel after 6 months. The results are shown in Tables 1 to 2 for about 20% and -20% adjustments in parameters.

Multi- Variable Optimization Test

In Tables 1 and 2 tests were carried out to find a feasible solution for the case when the error can occur through all parameters being contaminated (altered) simultaneously as the initial guess.

The optimization process for multi-variables consists of a series of single variable solutions. For instance, in the first iteration process, taking the initial guess of the first parameter, α, a solution can be obtained. This is continued for every single variable at every next iteration process. The solution obtained for the every series of the single variable has an affect on other parameters' values. In the multi-variables case, several local minima may be found, including the global minimum found as for the single variable cases. Based on the different values of the parameters involved, out of many feasible solutions available in a region, one of them would be the "optimum solution" when the multi-variable optimization converge.

In multi-variable technique, result shows that, when many parameters are disturbed, the NAG Routine takes the value of each parameter, calculates the bed level and compares the calculated bed level with the observed values. The difference between the two observed and calculated bed changes after each iteration process is the means for searching towards zero or a minimum solution, that possibly can be found. Finally, based on the relationship of variables, and effects of parameters on themselves as well as on the solution of governing equations, once a minima is found then optimization process can terminate.

Table 1, Multi-variable optimization, of parameters with -20% alterations

it. no	initial α, -20% away	initial α_1, -20% away	initial β_1, -20% away	initial γ_1, -20% away	initial Ta, -20% away	obj. function
1	0.028799	0.00800	3.1779	0.0799	0.0048299	0.36234573
10	0.035678	0.00996	4.1244	0.0999	0.0059950	0.00326783
20	0.035999	0.01000	4.0000	0.0999	0.0060001	0.00052020
50	0.035999	0.01010	4.0000	0.0999	0.0060001	0.00000002
68	0.036205	0.01000	4.0002	0.0999	0.0060000	0.00000002

Table 2, Multi-variable optimization, of parameters with 20% alterations

it. no	initial α, +20% away	initial α_1, +20% away	initial β_1, +20% away	initial γ_1, +20% away	initial Ta, 20% away	obj. function
1	0.043200	0.01200	4.8005	0.1199	0.00719	0.18663249
20	0.042634	0.01000	3.9999	0.0999	0.00714	0.12102335
30	0.035999	0.01000	4.0001	0.1002	0.00600	0.00006480
40	0.035999	0.01000	4.0000	0.1001	0.00600	0.00000002
41	0.035999	0.01000	4.0000	0.1002	0.00600	0.00000002
48	0.035999	0.01000	4.0001	0.0999	0.00600	0.00000002

The application of optimization is used here to check the best fit parameters values for a standard formula. The technique, its efficiency and usefulness, is also used for the parameters found in the developed sediment transport equation.

Discussion

In real rivers there are many factors which may influence the sediment discharge, grain sorting and bed armoring prediction and none of the available transport formulae can be used in all conditions with confidence. It may however be feasible to use a relatively simple method to model a given reach and to optimize its parameters. A straightforward formula therefore may be used to describe a complex situation if its parameters are adjusted by optimization, to account for complex variations in geometry etc.

A general form of the Manning equation is used for friction prediction. In this equation, the roughness parameter, n, is optimized.

For predicting the sediment load, grain sorting and bed armoring of graded sediment, it is possible to develop a modified sediment transport equation with help of a standard sediment transport formula such as the Ackers-White.

A simple relationship is developed in which the sediment parameters α_1, β_1 and γ_1 are identified and optimized. Also a simple grain sorting and armoring algorithm is used. The active layer in the armoring algorithm, which is effectively engaged in exchange with the water may vary with changes in hydraulic conditions, gradation and size of the bed material, and the length of the computational time step, etc. Therefore the active layer thickness is also treated as an optimization parameter in the numerical model.

From the sensitivity analysis, $\boldsymbol{\alpha_1}$ was shown to be affected by other parameters in the solution process significantly this was in relation with the dimension of $\boldsymbol{\alpha_1}$. It was seen that introducing an error by each of parameters $\boldsymbol{\beta_1}$ **,** γ_1, have simultaneous affects on the sediment transport rate, **Qs,** and as well as on the hydraulic parameters of the flow-sediment routing equations. Thus every parameter in the sediment transport equation can have their simultaneous affects on others, i.e., alteration of one parameter, causes alterations and errors on other parameters, producing the overall errors on the solution of the optimization process.

In the simple grain sorting and bed armoring algorithm developed by the authors [5], which is used in this study the active layer thickness is assumed to be equal to $\mathbf{d_{max}}$. It is effectively engaged in exchange with water, and also with variation in hydraulic conditions, gradation and size of the bed material, and the length of the computational time step, etc. The active layer thickness was found a sensitive parameter and was treated as an optimization parameter in the numerical model. For the case studied here, it was not sensitive as some of the others, in fact, orders of magnitude was less in the objective function.

An optimization technique based on the gradient method [NAG subroutine E04FDF [9] Gauss-Newton and modified Newton method was utilized to identify the true values of these parameters by iteration. The objective function used, based on bed level changes is found sensitive to all of the selected parameters. This equation is shown to be reasonably suitable for identification of the graded sediment routing parameters.

References:

1. Ackers, P., and White, W.R., “Sediment Transport-New Approach and Analysis,” Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY11, November, 1973, 2041-2060.

2. Akbari, G.H., Wormleaton, P.R., Ghumman, A.R., “A Simple Bed Armoring Algorithm for Graded Sediment Routing in Rivers”, Water for a Changing Global Community, 27th IAHR Congress, 10-15 August 1997 San-Fransisco, USA.

3. Akbari, G.H., “Optimizing resistance parameter for using real river data bed level calculations”, Journal Science and Engineering, USB, Vol 6, Summer 2004"

4. Akbari, G.H., "Sensitivity analysis of the most important sediment transport parameters in a sediment transport model", Journal Science and Engineering, USB, Vol 6, Summer 2004".

5. Akbari, G.H., “Fully coupled non-linear mathematical model for flow-sediment routing through rivers”, PhD Thesis, Q.M. University of London, 2003

6. Chin, C.O., Melville, B.W., Raudkivi, A.J. “Armoring Development” J. Hyd. Engg. ASCE, Vol. 120, No. 8, pp. 899-918, 1994.

8. Fisher, K.R. “Manual of Sediment Transport in Rivers” HR, Wallingford Report, May 1995.

7. Ghumman A. R, Wormleaton P.R, Hashmi H.N, **Akbari G.H**, "Parameter identification for Sediment Routing in Rivers" IAHR, Journal of Hyd. Research, Vol. 34-1996.

8. Little, W.C., Mayer, R.G., “The Role of Sediment Gradation on Channel Armoring”, Pub. No. ERC-0672, School of Civil Engg., Georgia Instit. of Technology, Atlanta, Georgia, 1972.

9. NAG, “NAG Fortran Library Manual”, QMW, Univ. of London, 1985.
10. Van Rijin, L.C., “Sediment transport”, Part 1,2, and 3, J. Hyd. Engg., ASCE, Vol. 110, pp. 1431-56, 1613, 41, and 1733-54, 1984.
11. White, W.R., and Day, T.J., “Transport of Graded Gravel Bed Material”, Gravel Bed Rivers, Hey, R.D., Bathurst J.C, and Thorne, C.R., ed., John Wiley & Sons, pp. 181-223., 1982.
12. Willetts, B.B., Maizels, J.K., and Florence, J. “The Simulation of Stream Bed Armoring and its Consequences”, Proc. Inst. of Civil Engrs, Part 1, Vol. 82 pp. 799-814, 1987.