Evaluation of the Procedure for Including Inertial Soil Structure Interaction in Nonlinear Static Analysis

I.Behmanesh[1] and F.Khoshnudian[2]
Amirkabir University of Technology

i_behmanesh@alum.sut.edu

Abstract

Effects of soil structure interaction (SSI) in elastic systems have been extensively studied in the past. It was introduced in some codes and provisions like NEHRP (1997, 2000, and 2003), ASCE 2005, and originally in ATC 3-06. However those did not consider yielding behavior of structure. The latter effect has attracted few investigations. Therefore, in this paper we investigate SSI effects in inelastic structures with comprehensive study for wide ranges of non-dimensional parameters to include SSI effects in Nonlinear Static Analysis Procedures (NSP).

To consider inelastic behavior of structures the last document considering SSI in inelastic structures, FEMA-440,defines an equivalent period lengthening ratio which alters the elastic period lengthening ratio by using ductility of the system to determine foundation damping from the known elastic formulation. In addition, in the whole procedure structure-soil system is replaced by rigid base SDOF system calculated by pushover analysis. Obviously that procedure cannot lead to exact responses; to evaluate it, we consider a SDOF structure with different periods changing from 0.2 to 3 seconds and different ductilities: 2, 4, and 6. This SDOF structure is located on the homogeneous half space defined by non-dimensional frequency (a0), aspect ratio (S), mass ratio and radius ration of the foundation.

The SDOF with idealized bilinear behavior is used as the structural model. The soil is modeled by fundamental lumped parameters based on the concepts of cone models representing soil with a 3DOF system. The proposed 4DOF system is analyzed in time domain using average acceleration method with several ground motions recorded on very soft soil sites.

First, this exact response is compared with the response of the inelastic rigid base SDOF system with initial and secondary stiffness determined by pushover analysis to evaluate the sufficiency of the basic concepts of FEMA-440: replacing fixed base structure and using of elastic damping values by equivalent period lengthening. It should be noted that the main procedure of FEMA-440 is checked too and the comparisons reveal an unsuitable estimations for exact responses, but it will not be discussed here because it has another source of error in equivalent linearization. So we use non linear analysis in calculating the response of replaced SDOF structure to achieve our own aims.

Next, the accuracy of two FEMA-440 methods for determining maximum displacement demands, Equivalent Linearization and Coefficient Method are evaluated with exact results.

Keywords: Soil Structure Inertial Interaction, Foundation Damping, Inelastic Structure, Equivalent Linearization, Coefficient Method.

Introduction

The effects of SSI have been the subject of numerous investigations in the past. However, they have been generally examined at the exclusion of the nonlinear behavior of the structure. For the elastic structure, the pioneers Veletsos and Meek (1974) [1], Veletsos and Nair (1975) [2] showed that the effects of inertial interaction could be approximated by increasing the fundamental period and changing the damping of a fixed base structure. Other authors studied about increased period and changed damping, Wolf (1985) [3], Aviles and Perez-Rocha (1996) [4] and (1999) [5].

However, yielding behavior of structure has recently been attracted some researches. Bielak [7] first studied in this context with the harmonic response of bilinear structure supported on a viscoelastic half-space. He stated that the resonant structural deformation could be significantly larger than would result if the supporting soil were

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1- M.Sc Student of Amirkabir University of Technology, Tehran, Iran.
2- Assistant Professor of Amirkabir University of Technology, Tehran, Iran.

rigid; nevertheless that could not clarify how soil effects affect the ductility demand or displacement demand of structure. It has been believed that yielding is a kind of energy dissipation process and thus leads to decrease the importance of interaction.

Aviles and Rocha (2003) [8] considered a SDOF elastic-plastic structure supported by a rigid foundation embedded in a viscoelastic stratum of constant thickness over a uniform viscoelastic half space. They assumed fixed parameters in order to approximate typical buildings and site condition of Mexico City. They introduced a replacement oscillator with effective period and damping ratio of elastic condition and effective ductility showing nonlinear behavior of structure. They have shown this simplified approach was accurate enough to estimate ductility and displacement demands by comparing with exact results.

They also found that the combined effects of foundation flexibility and structural yielding are beneficial for slender structures with natural period somewhat larger than the site period, but detrimental if the structural period is shorter than the site period. In [9] they used the described replacement oscillator formulation in the design code of Mexico City and NEHRP [10]. In addition, their investigation revealed that SSI can have a great influence on strength the reduction factor (SRF) and therefore can change the design process of the codes. To include SSI, [10] just find a new spectral response with effective period and damping without considering the changes in SRF values, leads to underestimate the design base shear.

In SSI systems, there are other sources of damping in addition to structural material damping, which are Radiation and material damping coming from the soil. These quantities are determined by dynamic properties of structure and soil. Nonlinear behavior of structure is defined by equivalent ductility, which is dependant on the ductility of structure and its dynamic properties. Equivalent period and damping are similar to elastic situations. However, as we will see, this definition is not enough, and to have better results, the damping definition of the replacement oscillator should contain the effects of nonlinear behavior of structure as well. Also the effect of post-yield stiffness ratio is evaluated for bilinear structures in this research.

Then, the accuracy of two FEMA-440 [11] methods for determining maximum displacement demands, Equivalent Linearization and Coefficient Method are evaluated with exact results.

Soil Structure Model used for exact results

We used a simplified model shown in Figure-1 to compute exact objective parameters. The structure is a bilinear SDOF system with stiffness K, period T, and post-elastic stiffness ratio α. m and h are lumped mass and height of the structure, which can be extended to the effective mass and height of MDOF structures. The mass moment of inertia labeled I. Moreover, the foundation is assumed as a circular disk.

The soil beneath the foundation is considered a homogeneous half space and is modeled by Fundamental Lumped Parameters base on concepts of Cone Models, extended by Wolf (1994) [12], representing the soil with a three DOF system. This model, with fixed parameters, is capable of including frequency dependency of dynamic soil stiffness. The dashpots and mass moment of inertia introduced in Figure-1 are defined as follows.

$$C_0 = \frac{r}{C_s}\gamma_0 K \qquad C_1 = \frac{r}{C_s}\gamma_1 K$$
$$M_0 = \frac{r^2}{C_s^2}\mu_0 K \qquad M_1 = \frac{r^2}{C_s^2}\mu_1 K \tag{1}$$

The parameter C_s is the degraded shear wave velocity which is consistent with the strain level of soil. The coefficient K of the springs is set equal to static-stiffness coefficient of the disk and four parameters γ_0, γ_1, μ_0, and μ_1 are listed below in Table-1 for two horizontal and rocking modes. They are set so as to achieve an optimum fit between the dynamic-stiffness coefficient of this model and the corresponding exact value of the disk on the soil half space by Veletsos and Verbic (1973) [13].

Table-1 Static stiffness and dimensionless coefficient of lumped parameter model for disk on homogeneous half space

Mode	Static Stiffness	γ_0	γ_1	μ_0	μ_1
Horizontal	$\frac{8Gr}{2-\nu}$	$0.78-0.4\nu$	–	–	–
Rocking	$\frac{8Gr^3}{3(1-\nu)}$	0.80	$0.42-0.3\nu^2$	$\begin{Bmatrix} \nu<\frac{1}{3} & 0 \\ \nu>\frac{1}{3} & 0.16(\nu-\frac{1}{3}) \end{Bmatrix}$	$0.34-0.2\nu^2$

G is the shear modulus and ν is Poisson's ratio of the soil.

The response of a soil structure system generally depends on the size of the structure, its dynamic properties, soil profile, and the applied excitation. The influence of these factors can be described by the following non-dimensional parameters:

1- A non-dimensional frequency as an index for structure to soil stiffness ratio defined as $a_0 = \frac{2\pi h}{TC_s}$, where T is the period of the structure in its fixed base condition. The practical range of a_0 for ordinary building type structures is from zero for the fixed base structures; to about three for cases with dominant SSI affects Ghannad et al (1998) [14].

2- Aspect ratio of the structure, defined as $s = \frac{h}{r}$.

3- Ductility demand of structure $\mu = \frac{u_m}{u_y}$, where u_m and u_y are the maximum displacement due to specific base excitation and the yield displacement, respectively.

4- Structure to soil mass ratio index, $\tilde{m} = \frac{m}{\rho r^2 h}$, where ρ is the unit weight of the soil. $\tilde{m}$ is taken to be 0.47 as recommended by Veletsos and Meek (1974).

5- The ratio of the mass of the foundation to that of the structure is defined as $\tilde{m}_f = \frac{m_f}{m}$. We assume it to be 0.1 in all parts of our analysis.

6- Poison's ratio of soil, ν, chosen 0.5, in that our time histories were recorded on soft soil sites.

7- Material damping of the soil and the structure. We set damping ratio of the structure to 5% as is usual, but the damping ratio of the soil to be zero.

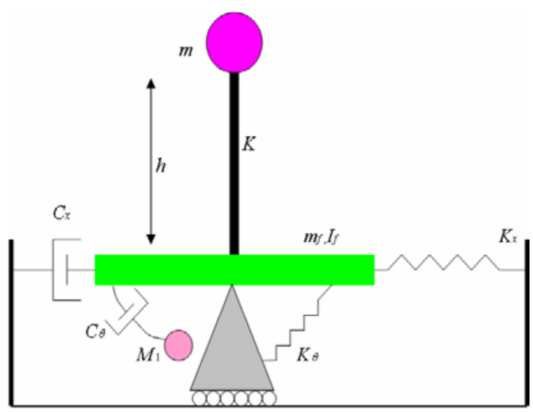

Figure-1 Soil-Structure System

Evaluation of Inelastic Damping Definition of FEMA-440

Nonlinear Replacement Oscillator

We employ this view with three different types of damping definition. The basic definition of the replacement oscillator considered here is based on [8]. They proposed a nonlinear replacement oscillator whose natural period and damping ratio are defined by the effective period and damping of the system rewritten below:

$$\tilde{T} = (T^2 + T_h^2 + T_\theta^2)^{1/2} \tag{2}$$

$$\tilde{\xi} = \xi \frac{T^3}{\tilde{T}^3} + \frac{\xi_x}{1+2\xi_x^2}\frac{T_x^2}{\tilde{T}^2} + \frac{\xi_\theta}{1+2\xi_\theta^2}\frac{T_\theta^2}{\tilde{T}^2} \tag{3}$$

In which:

$$T_x = 2\pi\left(\frac{m}{K_x}\right)^{1/2} \qquad T_\theta = 2\pi\left(\frac{mh^2}{K_\theta}\right)^{1/2} \tag{4}$$

are the natural periods if the structure were rigid and its base were only able either to translate or to rock, and

$$\xi_x = \frac{\pi C_x}{\tilde{T} K_x} \qquad\qquad \xi_\theta = \frac{\pi C_\theta}{\tilde{T} K_\theta} \tag{5}$$

To complete the model, equivalent ductility is also defined as follows:

$$\tilde{\mu} = (\frac{T}{\tilde{T}})^2 (\mu - 1) + 1 \tag{6}$$

Although the inelastic behavior of the structure is taken into account by relation (6), the other dynamic properties of the replacement oscillator, $\tilde{T}$, and $\tilde{\xi}$, are still defined in their elastic situation. As we will show, this kind of definition causes underestimation of the results.

In this study, we alter this error with the secant period or stiffness. We assume a SDOF with initial stiffness K, secondary stiffness αK, mass m, strength F_y, and material damping ξ.

If we put such a structure on a flexible foundation and perform pushover analysis, the load-deformation of the system will change like what is depicted in Figure-2. At yield moment, when the force in the structure is F_y, the force in horizontal spring of foundation is equal to it, and the moment in rocking stiffness of foundation is $F_y h$. Therefore, the yield displacement of replacement oscillator will be the summation of three series spring and can be calculated by relation (7). This situation remains after yielding too, but the difference is the value of stiffness of the structure that will be αK. With such a method, we can assemble the dynamic properties of replacement oscillator.

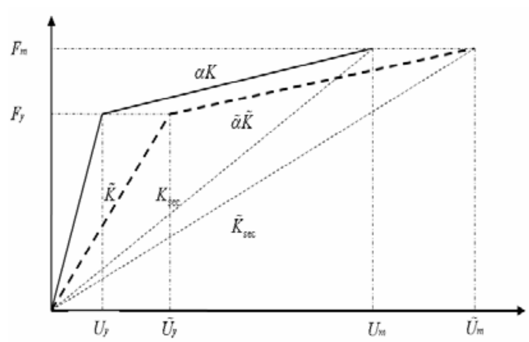

Figure-2 Load-Deformation curve of structure (solid line) and replacement oscillator systems (dashed line)

$$\tilde{U}_y = \frac{F_y}{K} + \frac{F_y}{K_x} + \frac{F_y h^2}{K_\theta} = F_y (\frac{1}{K} + \frac{1}{K_x} + \frac{h^2}{K_\theta}) = \frac{F_y}{\tilde{K}} \tag{7}$$

$$\tilde{\mu} = (\mu - 1)\Big[\frac{\tilde{K}}{K}(1-\alpha) + \alpha\Big] + 1 \tag{8}$$

$$\tilde{\alpha} = \alpha \frac{\mu - 1}{\tilde{\mu} - 1} \tag{9}$$

If we put $\alpha = 0$, the relation (8) will be the same as the equation derived for an elastic-plastic structure by [8], relation (6).

Relation (8) can support different values of post-yielding stiffness ratio, α, which was taken as zero previously. The belief is if the nonlinear structure with ductility μ can be replaced by an elastic structure with an equivalent period, we will be able to use the elastic definition of relation (3) to calculate equivalent damping of replacement oscillator. This equivalent period in this study is assumed to be the secant period, and set up as follows:

$$\tilde{T}_{sec} = \tilde{T} \sqrt{\frac{\tilde{\mu}}{1 + \tilde{\alpha}\tilde{\mu} - \tilde{\alpha}}} \tag{10}$$

$$\frac{\tilde{T}_{sec}}{T_{sec}} = \Big\{\frac{1}{\mu}\big[\frac{K}{\tilde{K}} - \alpha(\frac{K}{\tilde{K}} - 1) - 1\big] + \big[\alpha(\frac{K}{\tilde{K}} - 1) + 1\big]\Big\}^{1/2} \tag{11}$$

If $\tilde{T}_{\sec}$ and $\frac{\tilde{T}_{\sec}}{T_{\sec}}$ is used instead of $\tilde{T}$ and $\frac{\tilde{T}}{T}$ in relations (3) and (5), better results will be obtained. Therefore, relations (3) and (5) will change as described below below:

$$\tilde{\xi}_{eq} = \xi \frac{T_{\sec}^{\ 3}}{\tilde{T}_{\sec}^{3}} + \frac{\tilde{\xi}_x}{1+2\tilde{\xi}_x^2} \frac{T_x^2}{\tilde{T}_{\sec}^{\ 2}} + \frac{\tilde{\xi}_\theta}{1+2\tilde{\xi}_\theta^2} \frac{T_\theta^2}{\tilde{T}_{\sec}^{\ 2}} \tag{12}$$

$$\tilde{\xi}_x = \frac{\pi C_x}{\tilde{T}_{\sec} K_x} \qquad \tilde{\xi}_\theta = \frac{\pi C_\theta}{\tilde{T}_{\sec} K_\theta} \tag{13}$$

If we put $\alpha = 0$, the relation (11) will be almost similar to equation (8-8) of FEMA-440, rewritten below, relation (14)

$$\frac{\tilde{T}_{eff}}{T_{eff}} = \left\{ \frac{1}{\mu} \left[\frac{K}{\tilde{K}} - 1 \right] + 1 \right\}^{1/2} \tag{14}$$

However, FEMA-440 is using another damping definition rather than equation (3) derived by Veletsos and Nair (1975). In the following sections, we will show that this proposed damping type produces less error than FEMA-440 definition.

After determining the equivalent parameters of the replacement oscillator, which computes the maximum displacement of system (involving soil deformations) with less time-consuming analysis, the maximum relative displacement of the structure is calculated by:

$$U_m = \frac{\tilde{K}}{K} \frac{\mu}{\tilde{\mu}} \tilde{U}_m \tag{15}$$

METHOD of ANALYSIS

The two types systems explained in section 2 and 3 are analyzed in the time domain by direct step-by-step average acceleration method, Chopra (2001) [15], subjected to six ground motions recorded on soft soil sites. These time histories chosen from appendix C of FEMA-440 are listed below in Table-2.

Table-2 Ground motions recorded on very soft soil sites used in this study

Date	Earthquake name	Magnitude (Ms)	Station name	Station Number	Component (deg)	PGA (cm²)
10/17/89	Loma Prieta	7.1	Foster City (APEEL 1; Redwood Shores)	58375	90	277.6
10/17/89	Loma Prieta	7.1	Redwood City (APEEL Array Stn. 2)	1002 (USGS)	43	270.0
10/17/89	Loma Prieta	7.1	Oakland, Outer Harbor Wharf	58472	35	281.4
10/17/89	Loma Prieta	7.1	Oakland, Outer Harbor Wharf	58472	305	265.5
10/17/89	Loma Prieta	7.1	Oakland, Title & Trust Bldg. (2-story)	58224	180	191.3
10/17/89	Loma Prieta	7.1	Oakland, Title & Trust Bldg. (2-story)	58224	270	239.4

The computation continues until the strength of the structure leads the structure to the target ductility.

NUMERICAL RESULTS

In this section, we compare the results of the replacement oscillator with three kinds of damping definitions introduced in section 3 with exact results determined by section 2. the elastic definition derived by Aviles and Rocha (2003) is labeled in figures " Aviles-Rocha", the proposed model, which is the modified version of aforementioned definition is labeled " Modified Aviles-Rocha", and finally the results of FEMA-440 damping quantity is expressed in figures "FEMA-440".

In Figure-3 the comparison is made for elastic-plastic condition, $\alpha = 0$, for two ductilities 2, and 6. The results are the average error values of maximum relative displacement of structure for six ground motions. The error function of the other objective parameters like the strength of the structure is almost similar to what is depicted for U_m . The error function is defined by:

$$error = \frac{Oscillator\ Value - Exact\ Value}{Exact\ Value} \times 100 \qquad (16)$$

As shown, the proposed damping makes the results larger than the elastic definition and slightly less than the FEMA-440 definitions. Actually It is more accurate for low ductility ranges but indifferent in larger ductility values. Therefore, this definition alter the almost excessive damping of the elastic definition and somehow is better than FEMA-440 definition, as if reduces its conservative results. As can be seen, elastic-damping definition usually causes negative error values.

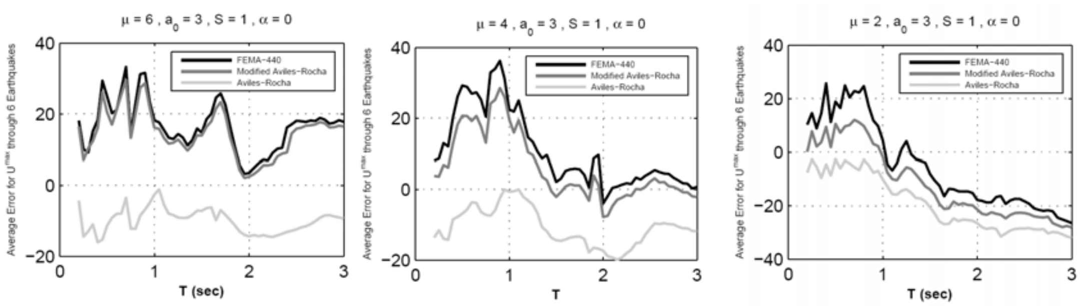

Figure-3 Average error values for relative structure displacement, $\alpha = 0$

Our results declare that with increase in slenderness, S , the errors would be subsided and also elastic damping would be acceptable. This is because of decrease in foundation damping in slender structure causing the results less sensitive to the type of the damping definition. However, speaking generally, the best choice through three definitions is the relation (12).

The other parameter which is not considered in FEMA-440 is the post-yield stiffness ratio. For six ground motions, analysis is done for post-yield stiffness ratios of $\alpha = 0.1\, and\ 0.2$. The average errors are shown in Figure 4 and 5 for U_m . As the aforementioned document does not consider the post yield stiffness ratio, α , the error values are more remarkable than the error values of the elastic-plastic assumption. Thus, proposed model is more accurate than FEMA-440 in its damping definition. This parameter is included in the analysis by relations (10) and (11).

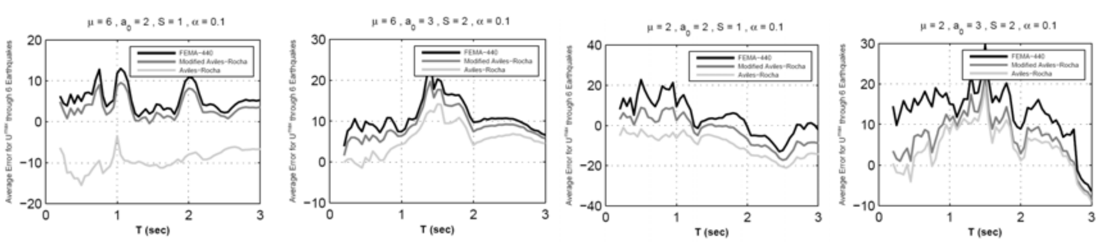

Figure-4 Average error values for relative structure displacement, $\alpha = 10\%$

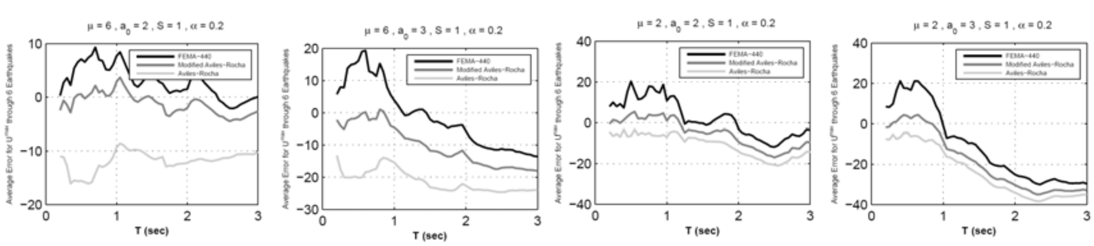

Figure-6 Average error values for relative structure displacement, $\alpha = 20\%$

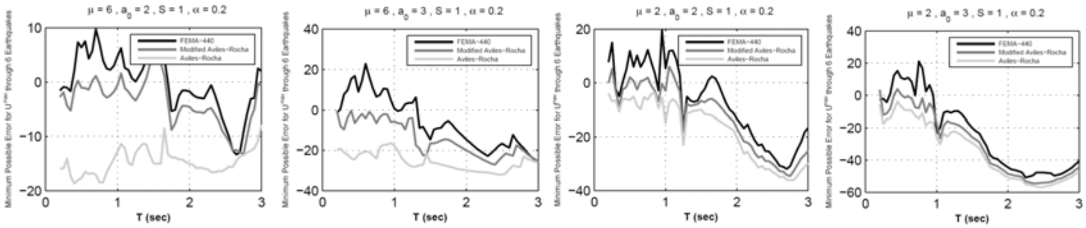

Figure-7 Minimum error values for relative structure displacement, $\alpha = 20\%$

Preference for using the relation (12) is intensified when comparing the minimum error values of the Figure-6 system in Figure-7. It can be seen that the error ranges decrease if we consider positive post-yield stiffness ratio. And the difference between FEMA-440 errors and Modified Aviles-Rocha is more remarkable rather than elastic-plastic structure's error. Therefore, in this case, using relation (12) leads to more reliable results.

Evaluation of Equivalent Linearization and Coefficient Method of FEMA-440 in Soil-Structure

In this section, for a specific ductility of the structure the exact system shown in Figure-1 is analyzed and the strength of the structure is obtained. This strength or yield force will be the yield force of a structure in corresponding period, a_0, and S that will be analyzed by the aforementioned two methods. It should be noted that in FEMA-440 procedures the given displacement and ductility demands are the total displacement (with soil displacement) and the ductility of the soil-structure system, not the structure. As a result, we use relation (15) to calculate structural demands from determined $\tilde{U}_m$ or $\tilde{\mu}$.

In Equivalent Linearization, period $\tilde{T}$ and damping ratio $\tilde{\xi}$ with proposed yield force characterize a SDOF equivalent fixed base system. In this method, some relations are proposed by FEMA-440 to convert a nonlinear SDOF to a linear one. The relations were obtained by regression analysis and by minimizing the difference between maximum displacement of fixed base nonlinear SDOF and corresponding equivalent linear system. As a result, those relations and their regression analysis might not be proper for Soil-structure system since Soil Structure Interaction causes remarkable effect in ductility demands of the structure (Aviles and Perez-Rocha (2005b), Ghannad and Ahmadnia (2006) [16]). In this view, if we are able to convert the soil-structure system to a nonlinear fixed base system, as discussed in previous title, the regression analysis of FEMA-440 would be usable.

Similar situation exists for Coefficient Methods, where the elastic displacement of fixed base SDOF is multiplies by some coefficient to determine inelastic displacement. One of the mentioned coefficients is inelastic displacement ratio, and it is equal to $\frac{\mu}{R}$ that is distinguished by C_R, for constant R, or C_μ, for constant μ. This parameter has two aspects in our investigation. First, this parameter, introduced by C_1 in FEMA-440, is determined by the relation valid mainly for firm sites. Thus, in soft soil sites, this coefficient should be altered by some proposed relations like Ruiz-Garcia and Miranda (2004) [17] and (2006) [18]. Second, since strength reduction factor is affected considerably by SSI, the mentioned relations cannot be used in soil-structure systems. However, by the concepts explained for Equivalent Linearization, this method could be extended for such system as well.

Figure-8 shows the response spectra of two different records, calculated by three methods.

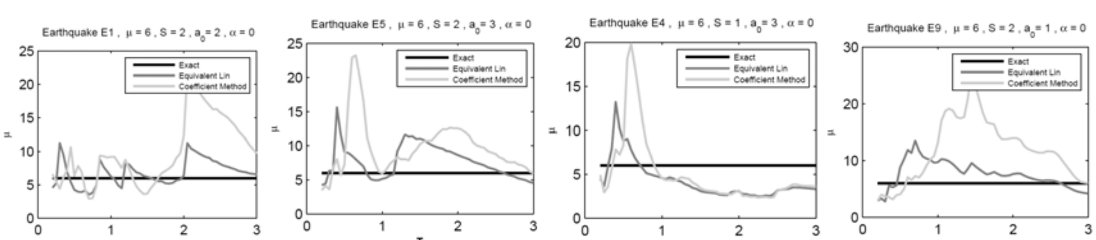

Figure-8 Comparison between Ductility response spectra; from left: Redwood City (APEEL Array Stn. 2); Foster City (APEEL 1; Redwood Shores); Emeryville (6363, Christie component 260°); Larkspur Ferry Terminal (comp 360°)

As can be seen in some period ranges, which are varied for four different ground motions, the responses are enormously conservative and unacceptable. This behavior is because of the fact first introduced by Miranda (1993). In soft soil sites, the C_1 values have their minimum in about the dominant ground motion period. These parameters for above records are 2.77 sec and 1.11 sec, 1.02 sec, and 1.51 sec respectively. the difference in these values and peak points of the graphs is the period lengthening through SSI effects. Thus, if we use C_1 values determined for firm sites, such minimum values do not participate in the analysis, and produce excessive errors.

However, average ductilities of 13 Loma Prieta earthquakes recorded on soft soil site (7 additional ground motions as well as what tabulated in table 2) exhibit in Figure 9 and Figure 10 for two ductility levels $\mu = 2 \& 6$, and varied values of a_0 and S .

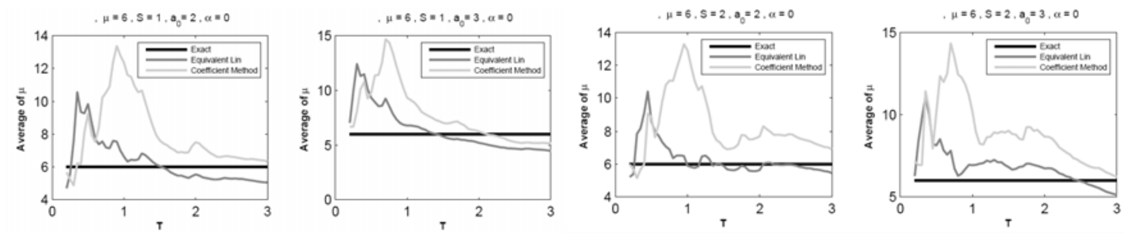

Figure-9 average structural ductility demands of 13 soft soil records; $\mu = 6$

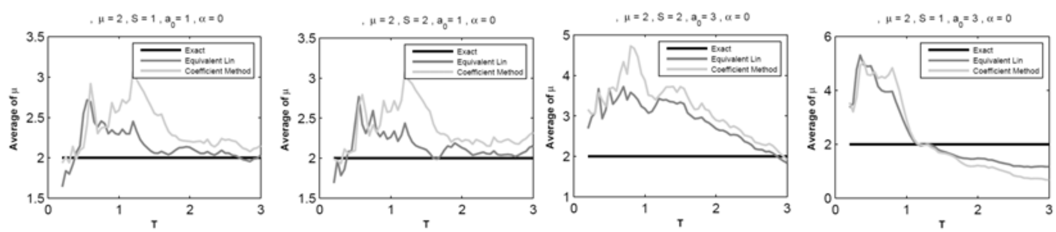

Figure-10 average structural ductility demands of 13 soft soil records; $\mu = 2$

Through 9 different configuration of a_0 and S , also 2 levels of ductility, we have seen that Equivalent Linearization overestimates the ductility and displacement demands considerably in low period ranges, but it is acceptable in long period ranges and the Coefficient Method overestimates the demands unacceptably in the most ranges of periods. Therefore, the relation for C_1 should be scaled by dominant period of motion and also by the period of soil-structure system, $\tilde{T}$.

It should be added that, in FEMA procedures, an approximate relation is introduced to determine response of the equivalent damped system (dampings are produced from foundation or nonlinear behavior of structure) from the response of 5% damping ratio elastic spectra. Nevertheless; elastic time history analysis is done to find those results. This leads to refrain from the errors coming from the approximate relation, and concentrates the errors on SSI effects.

Conclusion

The FEMA-440 definition of the foundation damping through the soil structure inertial interaction is evaluated and comparison is made between aforementioned damping ratio and damping ratio without considering nonlinear behavior of structure, defined by Aviles, J. and Perez-Rocha (1996), and its modified version, which is altered by the secant period of the structure and soil-structure system. Exact responses, determined by step by step average acceleration method of a 4 DOF simplified SSI model with several ground motions recorded on soft soil condition, are compared with nonlinear replacement oscillator response proposed by Aviles and Perez-Rocha (2003) with three damping definitions. The modified version contains the post-yield stiffness ratio effects. The results declare that the elastic damping formulation underestimates the demands, and the modified version, proposed here, is more accurate than FEMA-440 damping definition. Moreover, considering post-yield stiffness ratio, which is excluded in FEMA-440 procedures, can decrease the conservative results of elastic-plastic structures. Therefore, we propose that the equation (8-8) of FEMA-440 could be replaced by equation (11) in this paper. It should be noted that nonlinear time history analysis is performed in this evaluation. It means that none of simplified methods like Equivalent Linearization and Coefficient Method is used to refrain from their error sources.

Next, the procedure of Equivalent Linearization and Coefficient Method of FEMA-440 are investigated for soil-structure systems. The investigation declares that Equivalent Linearization gives conservative results in low structural period ranges, but acceptable results for medium and long structural period ranges. However, the Coefficient Method does not lead to proper results. This method overestimates enormously when the period of soil-structure system is close to predominant site period.

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