Structural Damage Localization by Modal Strain Energy Change

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Abstract
Structural damage detection, damage localization, based on modal strain energy change is presented in this research. If damage occurs in some members of a structure, stiffness of these members will be reduced and modal parameters; mode shapes and natural frequencies, in damaged structure are different from the undamaged state. The localization of damage is based on computing modal strain energy of each structural element in undamaged and damaged case, determining modal strain energy change so modal strain energy change ratio (MSECR) for each member which members with higher MSECR are suspected to be damaged. Only analytical mode shapes and structural matrices are required in this damage localization approach.
Keywords: Damage detection, Modal Analysis, Modal strain energy.

Introduction

All load-carrying structures such as buildings, bridges, aircrafts, spacecrafts and offshore platforms continuously accumulate damage -which can be defined as a change in physical properties of members- during their service life. When damage is occurs in one or more members of a structure, the stiffness and the load-carrying capacity of these members will be decreased thus the modal characteristics of undamaged and damaged structure are not same. Based on changes in frequencies, mode shapes, or their combinations, several structural damage detection techniques have been proposed and many researchers have studied the possibility of detecting the location and the magnitude of damage in recent years [1].

The methods for damage identification are commonly classified into four levels: Level 1: Determination that damage is present in the structure, Level 2: determination of the geometric location of the damage, Level 3: quantification of the severity of the damage, Level 4: prediction of the remaining service life of the structure. To date, vibration-based damage identification methods that do not make use of some structural model primarily provide Level 1 and Level 2 damage identification. When vibration-based methods are coupled with a structural model, Level 3 damage identification can be obtained in some cases. Level 4 prediction is generally associated with the fields of fracture mechanics, fatigue-life analysis, or structural design assessment [8].

Shi et al. (1998) proposed a method to detect the location of damage using the elemental energy quotient difference from incomplete and noisy measured modal data [2]. Shi et al. (2000) using a method based on modal strain energy change to structural damage detection. In the proposed method the location of damage determined based on computation of modal strain energy for each structural element and by use of iterative method damage severity in any damaged member can be determined [3]. Barroso and Rodriguez (2004) proposed a methodology based on ratios between stiffness and mass values from the eigenvalue problem to identify the undamaged state of the structure and using the damage index method to detect the location and severity of damage. This approach may be applicable to existing structures that may already incorporate some amount of damage [4]. Escobar et al. (2004) using the geometrical transformation matrix to obtain the condensed stiffness matrix of a structure that can be estimated for the damaged state, and with an iterative scheme estimate the condensed stiffness matrix of non-damaged state of the structure that it is possible to detect structural damage by comparing changes on structural dynamic parameters with their defining properties [5]. Stubbs et al. (2005) introduce a new form of damage index

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based on the changes in the distribution of the compliance of the structure due to damage. In this method the changes in the compliance distribution are obtained using the mode shapes of the pre-damaged and the post-damaged state of the structure [6]. Huynh and Tran (2005) using frequency response functions that obtained from non-destructive vibration tests to structural damage detection. In this method change in structural stiffness matrix is reflected in changes of frequency response function data which can be exemplified by the evaluation of damage location vector that requires the dynamic stiffness matrix of the undamaged structure and the change in frequency response functions of the currently damaged structure [7].

Theory

Structural damage often causes a loss of stiffness in one or more elements of a structure, but not a loss in the mass. In the theoretical development assumed that the damage only affects the stiffness matrix of the system. When damage occurs in the structure, it can be represented as a small perturbation in the original system. Thus, the stiffness matrix K^d, the *ith* modal eigenvalue λ_i^d and the *ith* mode shape Φ_i^d of the damaged structure can be expressed as [2]:

$$K^d = K + \sum_{n=1}^{N_d} \Delta K_n = K + \sum_{n=1}^{N_d} \alpha_n K_{l_n} \qquad (-1 < \alpha_n \le 0) \tag{1}$$

$$\lambda_i^d = \lambda_i + \Delta\lambda_i; \quad \Phi_i^d = \Phi_i + \Delta\Phi_i \tag{2}$$

The superscript *d* denotes the damage case, N_d is total number of the damaged members, α_n is damage extent that is expressed as a fractional change of the elemental stiffness matrix and l_n is the element number of the *nth* damaged element, respectively [2].

Denoting M and K as the mass and stiffness matrices for the undamaged structure model, in the eigenanalysis for the baseline structure one writes:

$$K\Phi_i = \lambda_i M \Phi_i \tag{3}$$

Likewise, corresponding expression for the damaged structure as:

$$K^d \Phi_i^d = \lambda_j^d M^d \Phi_j^d \tag{4}$$

Where M^d is mass matrices for the damaged structure, λ_j^d and Φ_j^d denote the associated *jth* eigenvalue and eigenvector in damaged structure. In Eqs. (3) and (4), Φ_i and λ_i, Φ_j^d and λ_j^d are the analytical modal information for the undamaged and damaged structure [3].

Modal Strain Energy Change

The elemental modal strain energy (*MSE*) is defined as the product of the elemental stiffness matrix and the second power of the mode shape component [2]. For the *jth* element and the *ith* mode, the *MSE* before and after the damage defined as [3]:

$$MSE_{ij} = \Phi_i^T K_j \Phi_i; \; MSE_{ij}^d = \Phi_i^{d\,T} K_j \Phi_i^d \tag{5}$$

where MSE_{ij} and MSE_{ij}^d are undamaged and damaged *MSE* of the *jth* element for the *ith* mode shape, respectively. Because the damage elements are not known, the undamaged elemental stiffness matrix K_j is used instead of the damaged, one as an approximation in MSE_{ij}^d [3].

The modal strain energy change (*MSEC*) of the *jth* element for the *ith* mode could be obtained from:

$$MSEC_{ij} = \Phi_i^{d\,T} K_j \Phi_i^d - \Phi_i^T K_j \Phi_i \tag{6}$$

similarly the modal strain energy change ratio (*MSECR*) defined as follows:

$$MSECR_{ij} = \frac{\left| MSE_{ij}^d - MSE_{ij} \right|}{MSE_{ij}} \tag{7}$$

where *j* and *i* denote the element number and mode number, respectively [3]. The *MSECR* has been verified to be a good indicator for damage localization [2].

In the structure $MSECR_{ij}$ is calculated for all elements. If more than one measured mode is available, $MSECR_{ij}$ is calculated for all the modes, and $MSECR_j$ of the *jth* element is defined as the average of summation of all $MSECR_{ij}$ normalized with respect to the largest value of $MSECR_{ij}$ for each mode [3]:

$$MSECR_j = \frac{1}{m}\sum_{i=1}^{m} MSECR_{ij} \Big/ MSECR_{\max} \tag{8}$$

where *m* is total number of measured mode, respectively. The location of damage can be identified by examining those values of $MSECR_j$ that are larger than the others [2].

Numerical Studies

Numerical examples consists of two dimensional, single bay, three story moment resisting steel braced frame with one: sixth scale of the existing jacket platform in south California. The finite element model of this frame consists of 23 elements with 13 nodes and 35-DOFs as shown in figure.1. Yield stress of the horizontal and vertical bracing is 36ksi and in the leg elements is 47ksi, modulus of elasticity and mass density of steel is 29000ksi and 2.830e-4 kip/in^3. The bay width is 120in and distance between the horizontal bracing from end supports are 30, 150, 270 and 330in. Geometrical information of frame elements except of elements 22, 23 are listed in tables. 1 and 2 [9].

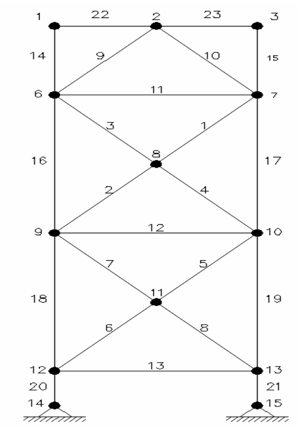

Figure 1–elements and joints numbers of numerical studies

Elements 22, 23 are used to model the deck of offshore platform with 2W24*55 section [9].

Table 1- Geometrical dimension of frame elements (frame 1)

Element number	O.D., in	Thk, in
1,2,3,4,11,12,13	4	0.083
5,6,7,8	5	0.120
9,10	6	0.125
14,15,16,17,18,19,20,21	12 3/4	0.281

Table 2- Geometrical dimension of frame elements (frame 2)

Element number	O.D., in	Thk, in
1,2,3,4,11,12,13	4	0.120
5,6,7,8	4.5	0.188
9,10	6	0.181
14,15,16,17,18,19,20,21	12 3/4	0.375

Frames 1, 2 are modeled in the finite element software, OPENSEES, modal analysis has been performed and first mode shape of vibration of these frames is shown in figure.2.

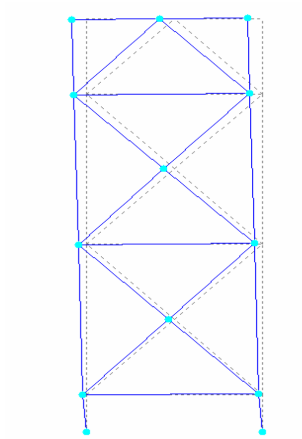

Figure 2– first mode of vibration (T=0.071s for frame1 and T=0.065s for frame 2)

Damage Localization

Frames 1, 2 are tested in the University Of Berkeley, California, to investigating cyclic inelastic behavior of steel offshore structure. Along the test of the frame 1; damage has occurred at elements 2, 3, 8 and 6, for the frame 2; at elements 2, 4, 5 and 8, respectively [9]. Therefore six damage cases are considered as the following:

Case 1, considers single-damaged element, element 2 of the frame 1 has been removed.
Case 2, considers single-damaged element, element 2 of the frame 2 has been removed.
Case 3, considers two-damaged elements, elements 2, 3 of the frame 1 have been removed.
Case 4, considers two-damaged elements, elements 2, 4 of the frame 2 have been removed.
Case 5, considers three-damaged elements, elements 2, 3 and 8 of the frame 1 have been removed.
Case 6, considers three-damaged elements, elements 2, 4 and 5 of the frame 2 have been removed.

Results of modal analysis in each damage case are listed in table.3.

Table 3- first period of vibration in any damage case

Period, s					
Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
0.0770	0.0708	0.1034	0.0960	0.1209	0.1121

Location of damage is detected by computing modal strain energy for each structural element with respect to first mode of vibration and results shown in figures 3-8. It should be noted that in this method, the elements with higher MSECR are suspected damaged elements [2].

As shown in figures 3, 4 results indicate that for the single-damage element, elements 1, 2, 3, 4, 9, 12, 14, 15, 16, 22 and 23 are suspected damaged elements but damage is most possible in element 2 for both of the frames 1, 2.

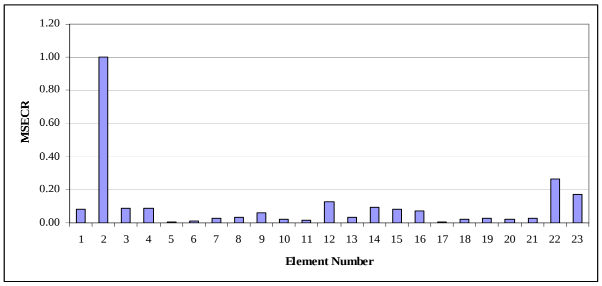

Figure 3– damage at element 2 of the frame 1

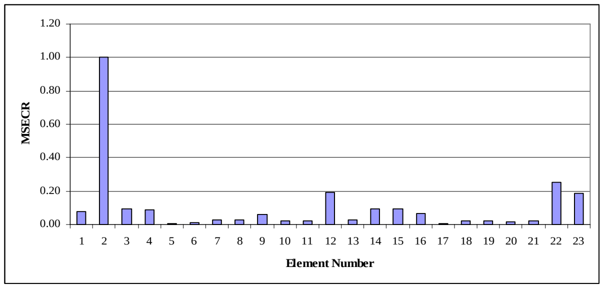

Figure 4– damage at element 2 of the frame 2

As shown in figure 5, results indicate that for the two-damage elements, elements 2, 3, 11, 12, 14, 15, 16 and 17 are suspected damaged elements but damages are most possible in elements 2, 3 of the frame 1. Similarly from figure 6, results indicate that for the two-damage elements, elements 2, 4, 11, 12, 14, 15, 16 and 17 are suspected damaged elements but damages are most possible in elements 2, 4 of the frame 2.

According to figure 7, results indicate that for the three-damage elements, elements 2, 3, 8, 11, 12, 14 and 15 are suspected damaged elements but damages are most possible in elements 2, 3 and 8 of the frame 1 and With respect to figure 8; results indicate that for the three-damage elements, elements 2, 4, 5, 11, 12, 14 and 15 are suspected damaged elements but damages are most possible in elements 2, 4 and 5 of the frame 2.

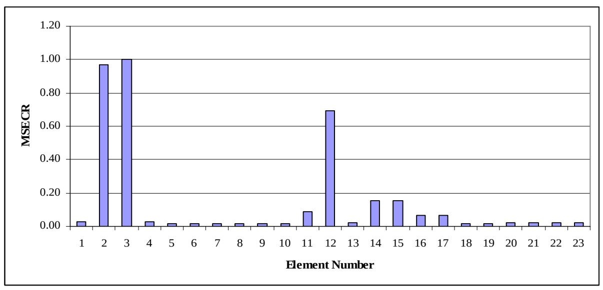

Figure 5– damage at elements 2, 3 of the frame 1

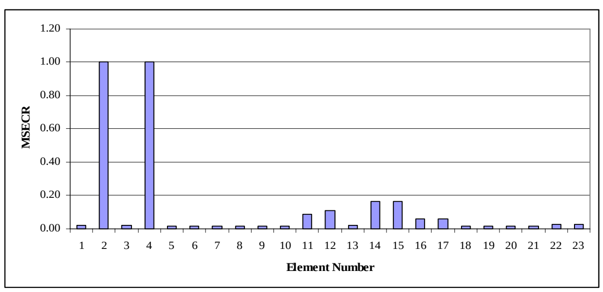

Figure 6– damage at elements 2, 4 of the frame 2

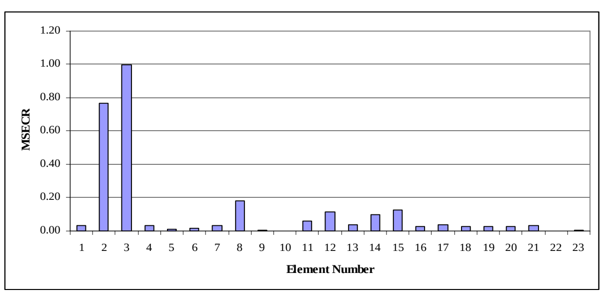

Figure 7– damage at elements 2, 3 and 8 of the frame 1

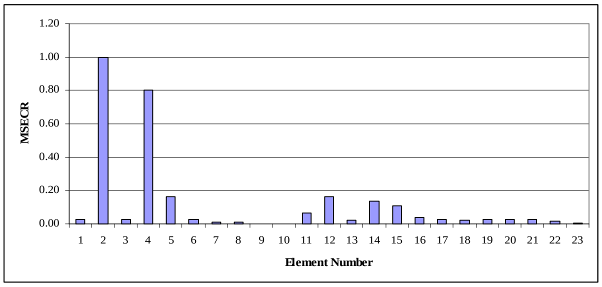

Figure 8– damage at elements 2, 4 and 5 of the frame 2

Concluding Remarks

1) The damage would alter the elemental modal strain energy in a structure.
2) Modal strain energy change ratio (MSECR) in damaged element(s) is more than other elements.
3) With increasing the primary damaged elements in a structure, number of suspected damaged elements will be decreased.
4) Due to occurring damage at vertical bracing of the special story, horizontal and vertical bracing and leg elements of that story are suspected damaged elements.

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